TSDOS

TRANSMISSION & SUBSTATION DESIGN & OPERATION SYMPOSIUM

THE UNIVERSITY OF TEXAS AT ARLINGTON

DEVELOPMENT OF ANALYTICAL METHODS FOR SPLICED LEG MEMBERS

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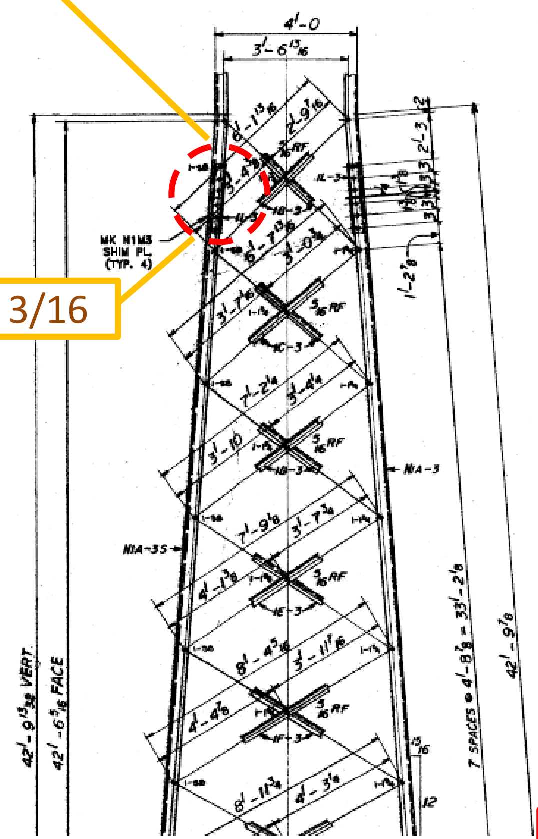
LOWER COLORADO RIVER AUTHORITY (LCRA)

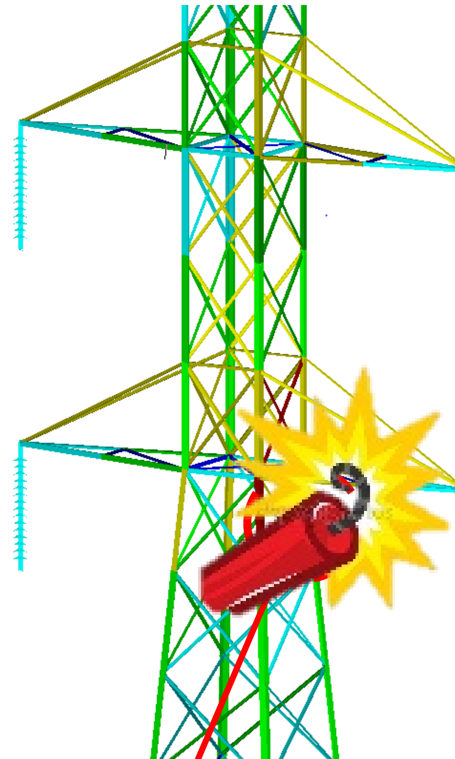
CONTENTS

I. ISSUES : Capability of PLS Tower

3 1/2 x 3 1/2 x 1/4

4 x 4 x 3/16





Modelling with
3 1/2 x 3 1/2 x ¼ member

I. ISSUES : Consequences

MODELLING A CONSTANT SECTION BASED ON THE SMALLER SECTION

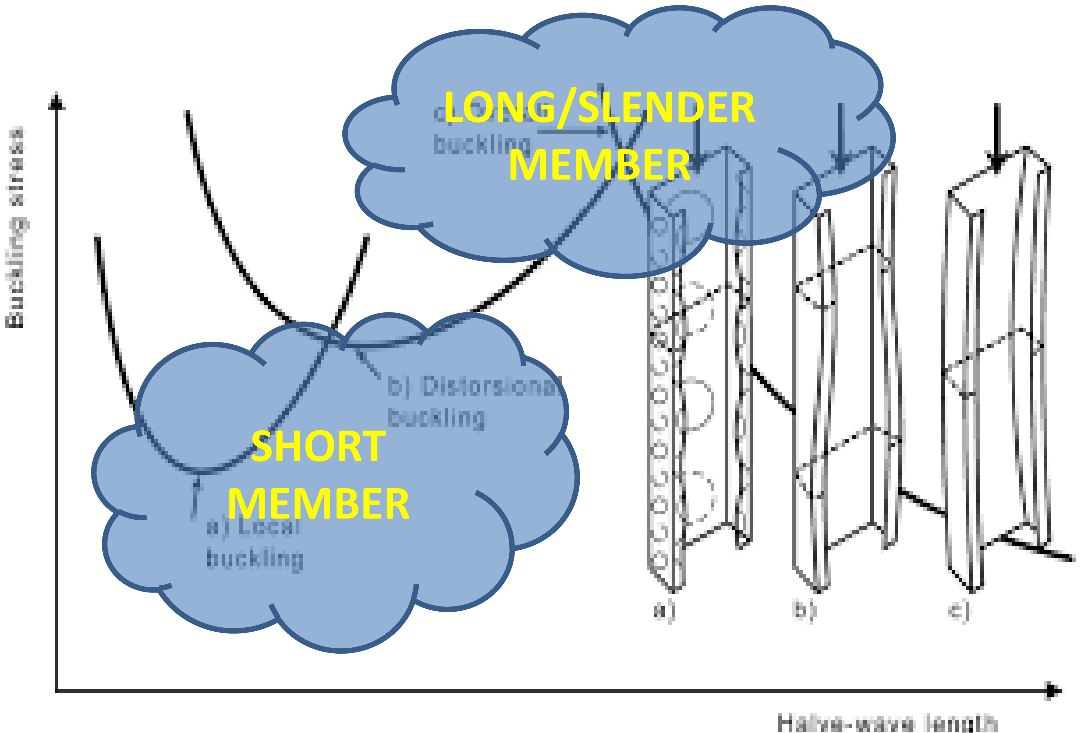
↓

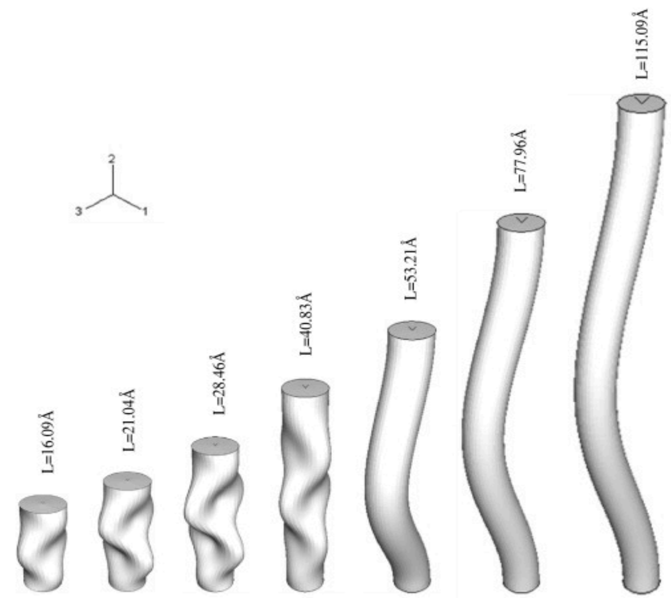
OVERDESIGN OF INDIVIDUAL MEMBER

OVERESTIMATION OF OVERALL STRUCTURAL BEHAVIOR

II. BACKGROUND THEORY : Compression Failure

*The failure modes of thin plate structures under the compression load can be categorized into the three modes which are **local, distorsional, and overall buckling**. They are mainly affected by member **slenderness ratio and fixity**.*





ENERGY • WATER • COMMUNITY SERVICES

Forces

(N, S, M, T)

$\propto$

Deformations

(δ, λ, θ, Φ)

$$[P] = [K][\Delta]$$

$$[P] = [M][\ddot{\Delta}] + [C][\dot{\Delta}] + [K][\Delta]$$

$$[\sigma] = [E][\varepsilon]$$

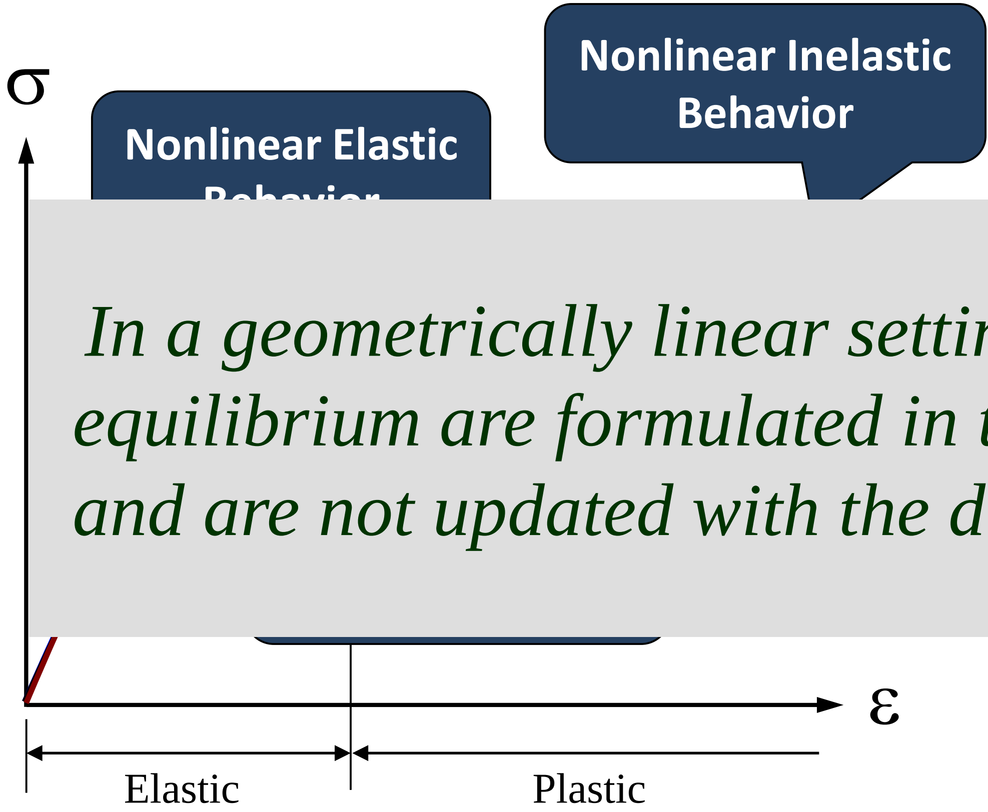
Stresses

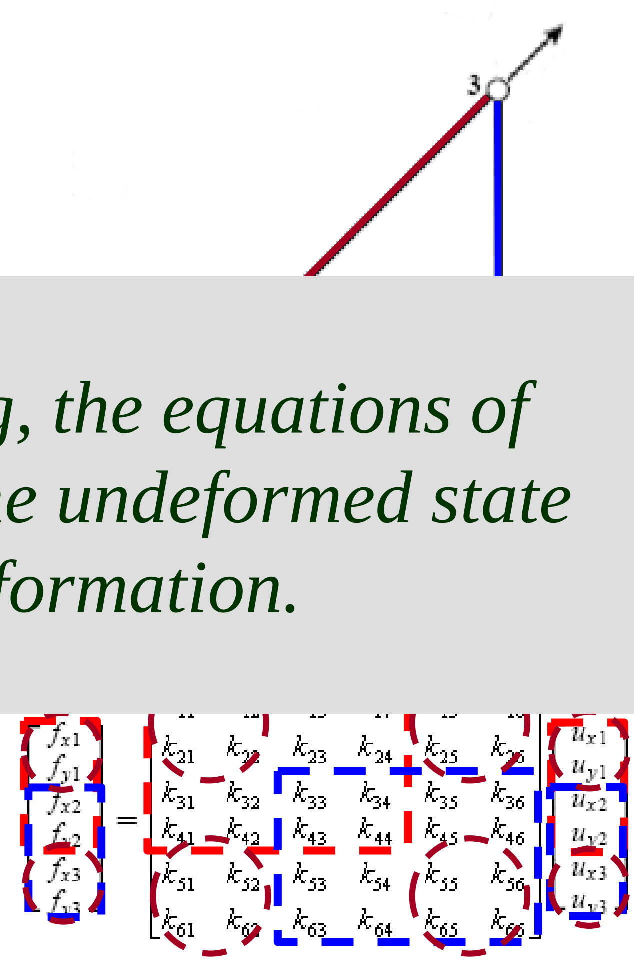
$(\sigma, \tau, \sigma_b, \tau_t)$

$\propto$

Strains

$(\varepsilon, \gamma, \varepsilon_b, \gamma_t)$





In a geometrically linear setting, the equations of equilibrium are formulated in the undeformed state and are not updated with the deformation.

LCRA ENERGY • WATER • COMMUNITY SERVICES

COLUMN	DESIGN COMPRESSION, F_a
SHORT ($KL/r \le C_c$)	$\left[1-\frac{1}{2}\left(\frac{KL/r}{C_c}\right)^2\right]F_{cr}$
SLENDER ($KL/r > C_c$)	$\frac{\pi^2 E}{(KL/r)^2}$ $\left(r=\sqrt{I/A}\right)$

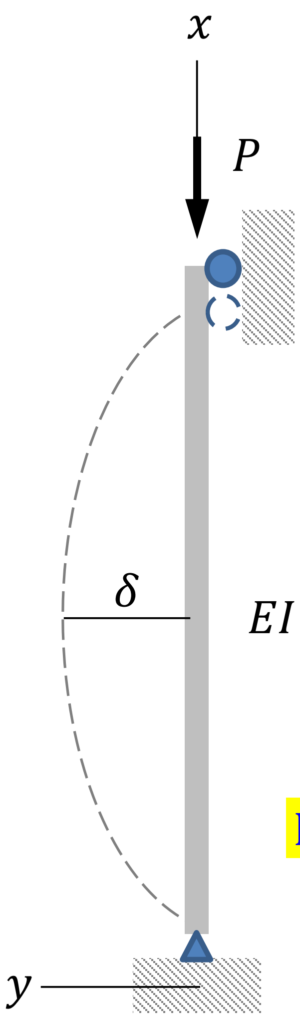
$$C_c = \pi\sqrt{\frac{2E}{F_y}}$$

Can be replaced with F_{cr} according to w/t ratio

CRITERIA	F_{cr}
$\frac{w}{t} < \frac{80}{\sqrt{F_y}}$	F_y
$\frac{80}{\sqrt{F_y}} \le \frac{w}{t} \le \frac{144}{\sqrt{F_y}}$	$\left[1.677 - 0.677\frac{w/t}{(w/t)_{lim}}\right]F_y$
$\frac{144}{\sqrt{F_y}} < \frac{w}{t}$	$\frac{0.0332\,\pi^2 E}{(w/t)^2}$

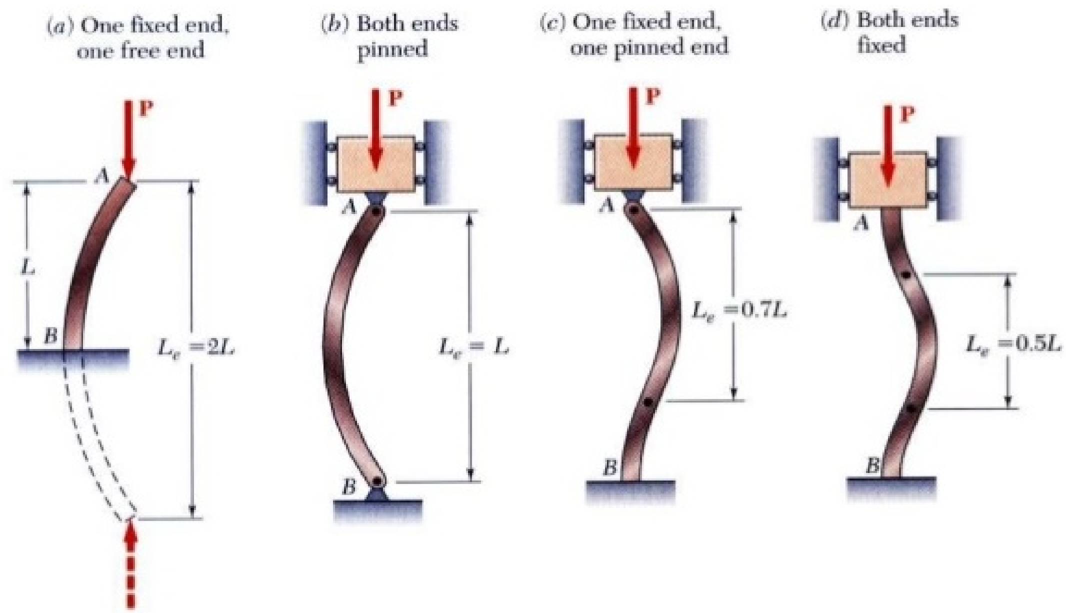
$$\left(\frac{w}{t}\right)_{lim} = \frac{80}{\sqrt{F_y}}$$

II. BACKGROUND THEORY : Slender (Euler's Formula)

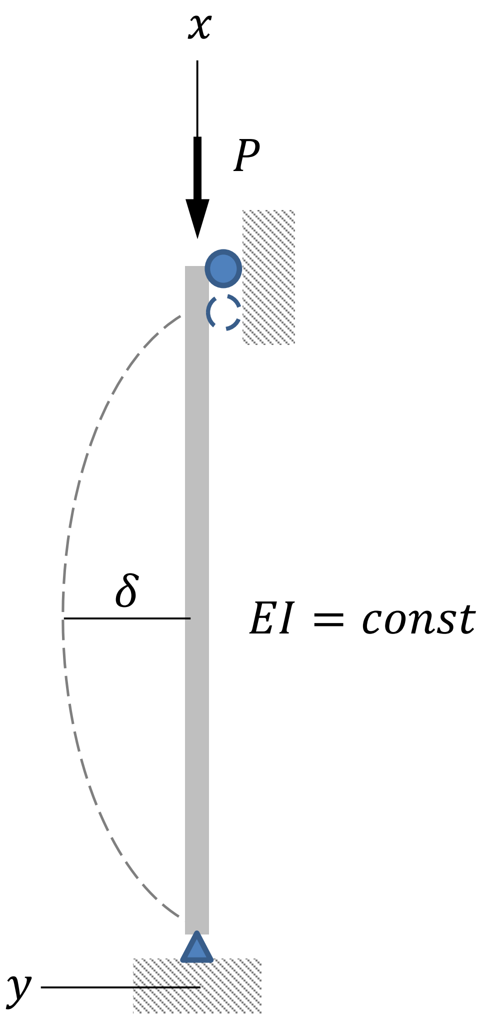


$EI = const$

$EIy'' = -Py$



B.C	P_{cr}	L_e (=KL)
Pin-Pin	$\pi^2 EI/L^2$	L
Fix-Free	$\pi^2 EI/4L^2$	2L
Fix-Fix	$4\,\pi^2 EI/L^2$	0.5L
Fix-Pin	$2.046\,\pi^2 EI/L^2$	0.699L



- Approximate solution
- Equilibrium condition between the strain energy of bending (ΔU) and the work done by the compressive force P (ΔT)

$$\Delta U = \Delta T$$

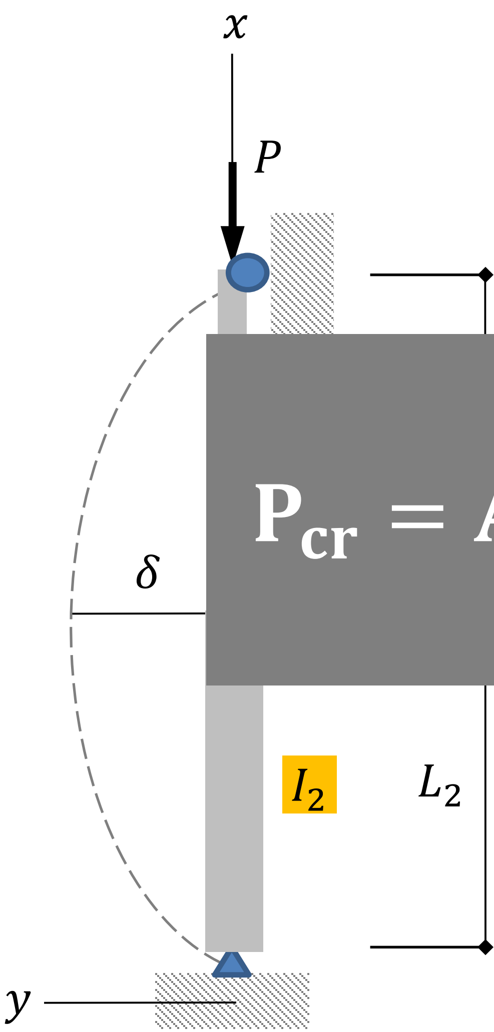
$$\Delta U = \int_0^l \frac{M^2}{2EI}\,dx$$

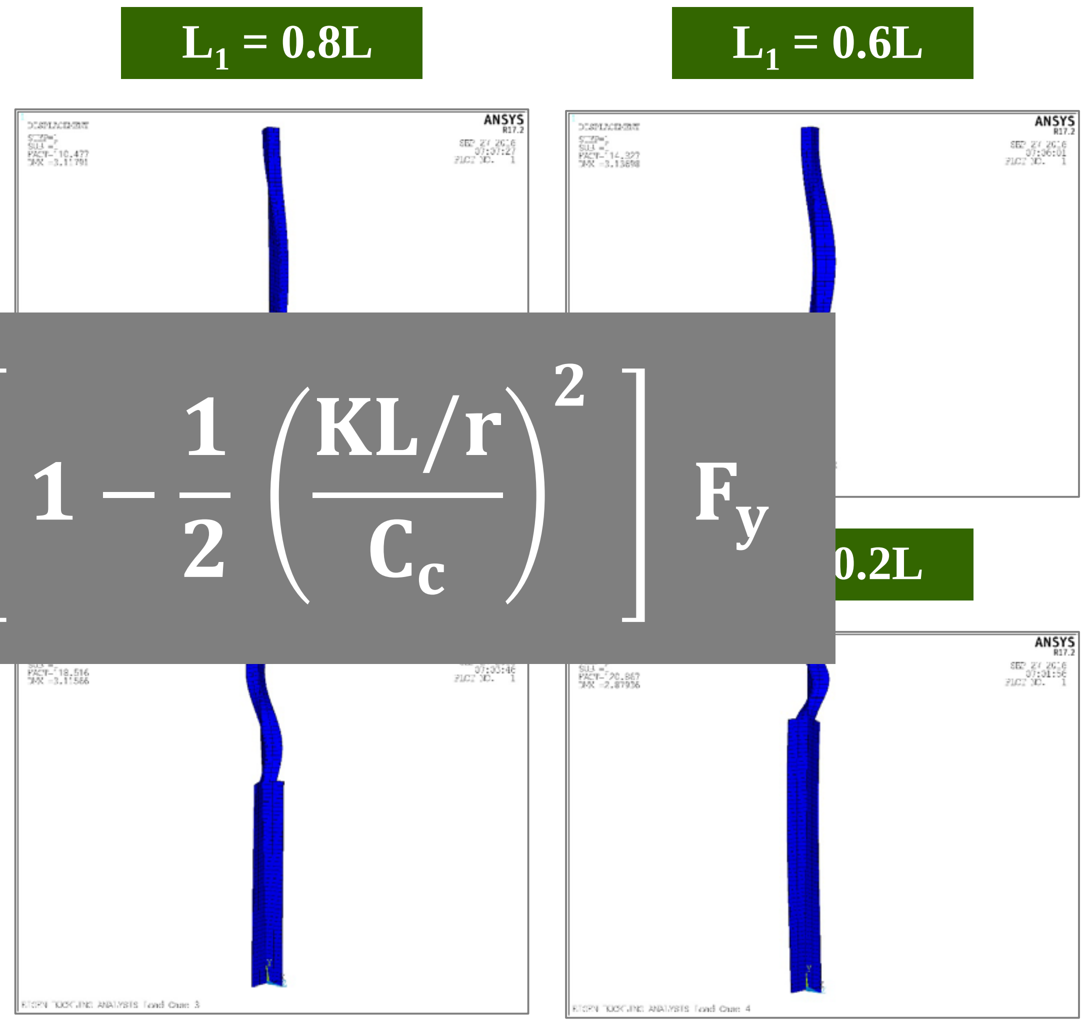
$(M = Py)$

$$\Delta T = P\lambda$$

$$\left(\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx \right)$$

B.C	DEFLECTION, y
Pin-Pin	$\sin(\pi x/l)$
Fix-Free	$\delta\left(1-\cos\frac{\pi x}{2l}\right)$
Fix-Fix	$\frac{\delta}{2}\left(1-\cos\frac{2\pi x}{l}\right)$

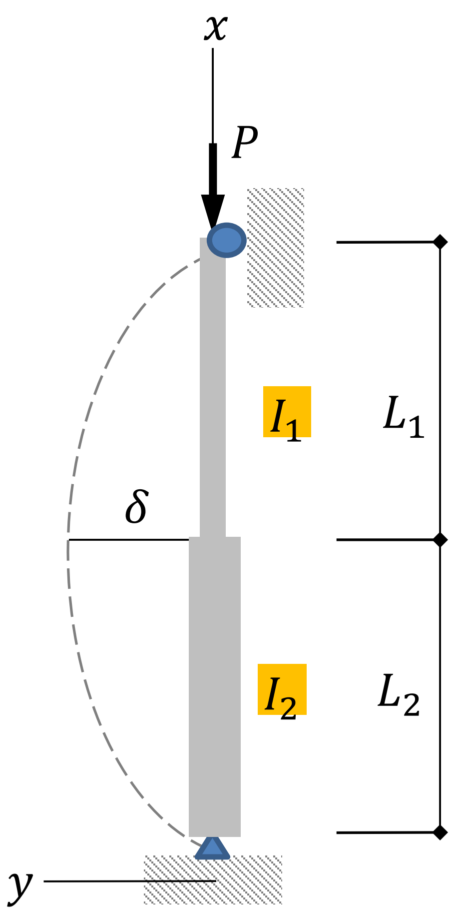




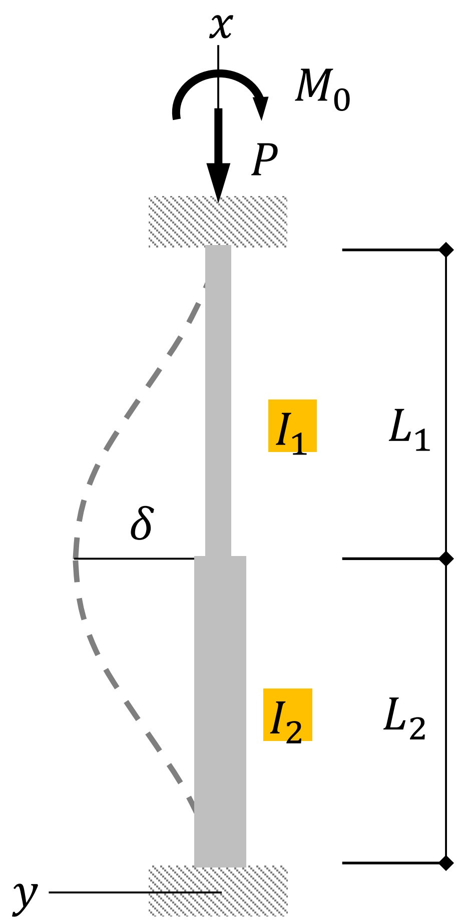
$$P_{cr} = A_1 \left[1 - \frac{1}{2} \left(\frac{KL/r}{C_c} \right)^2 \right] F_y$$

III. GOVERNING EQUATION : Slender Member

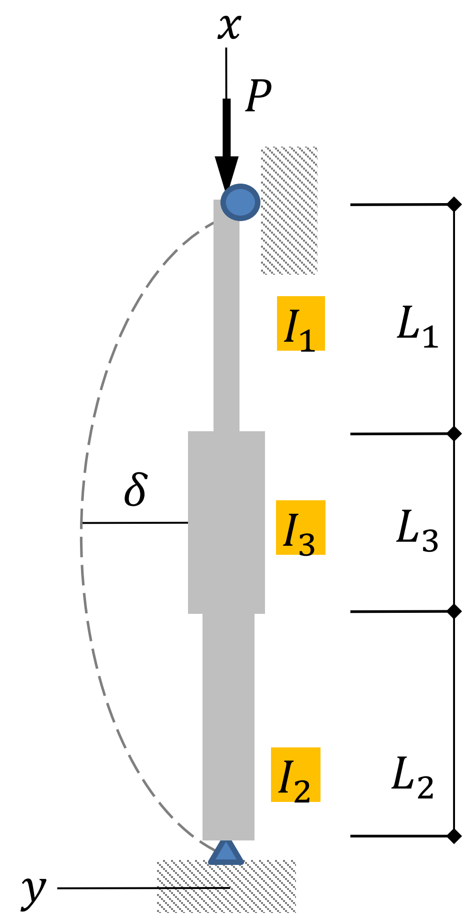
2 SEG. : P-P B.C



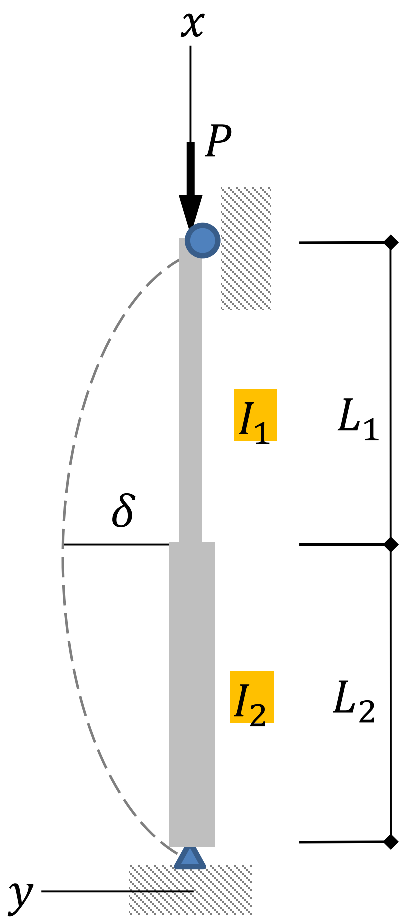
2 SEG. : F-F B.C



3 SEG. : P-P B.C



III. GOVERNING EQUATION : Slender (Euler's Formula)



$$EI_1 y_1'' = -Py_1 \quad\Rightarrow\quad y_1'' + k_1^2 y_1 = 0$$

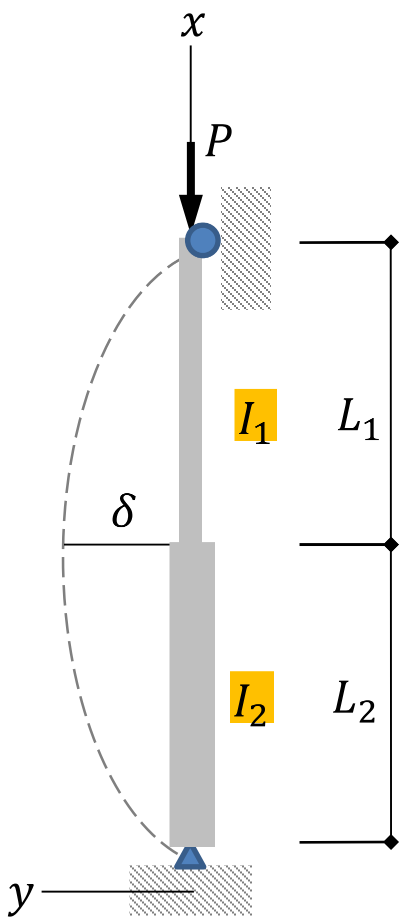
$$EI_2 y_2'' = -Py_2 \quad\Rightarrow\quad y_2'' + k_2^2 y_2 = 0$$

$$\begin{pmatrix} k_1^2 = P/EI_1 \\ k_2^2 = P/EI_2 \end{pmatrix}$$

$$y_1 = A\cos k_1 x + B\sin k_1 x$$
$$y_2 = C\cos k_2 x + D\sin k_2 x$$

B.C & I.C : $y_2(0) = y_1(L) = 0 \qquad y_2(L_2) = y_1(L_2) = \delta$

$$\frac{k_1}{k_2} + \frac{\tan k_1 L_1}{\tan k_2 L_2} = 0$$



$$\Delta U = \Delta T$$

Deflection Curve

$$y = \delta \sin(\pi x / l)$$

Strain Energy

$$\Delta U = \int_0^{l_2} \frac{M^2}{2EI_2} dx + \int_{l_2}^{l} \frac{M^2}{2EI_1} dx$$

Work

$$\Delta T = \frac{P}{2} \int_0^{l} \left(\frac{dy}{dx}\right)^2 dx$$

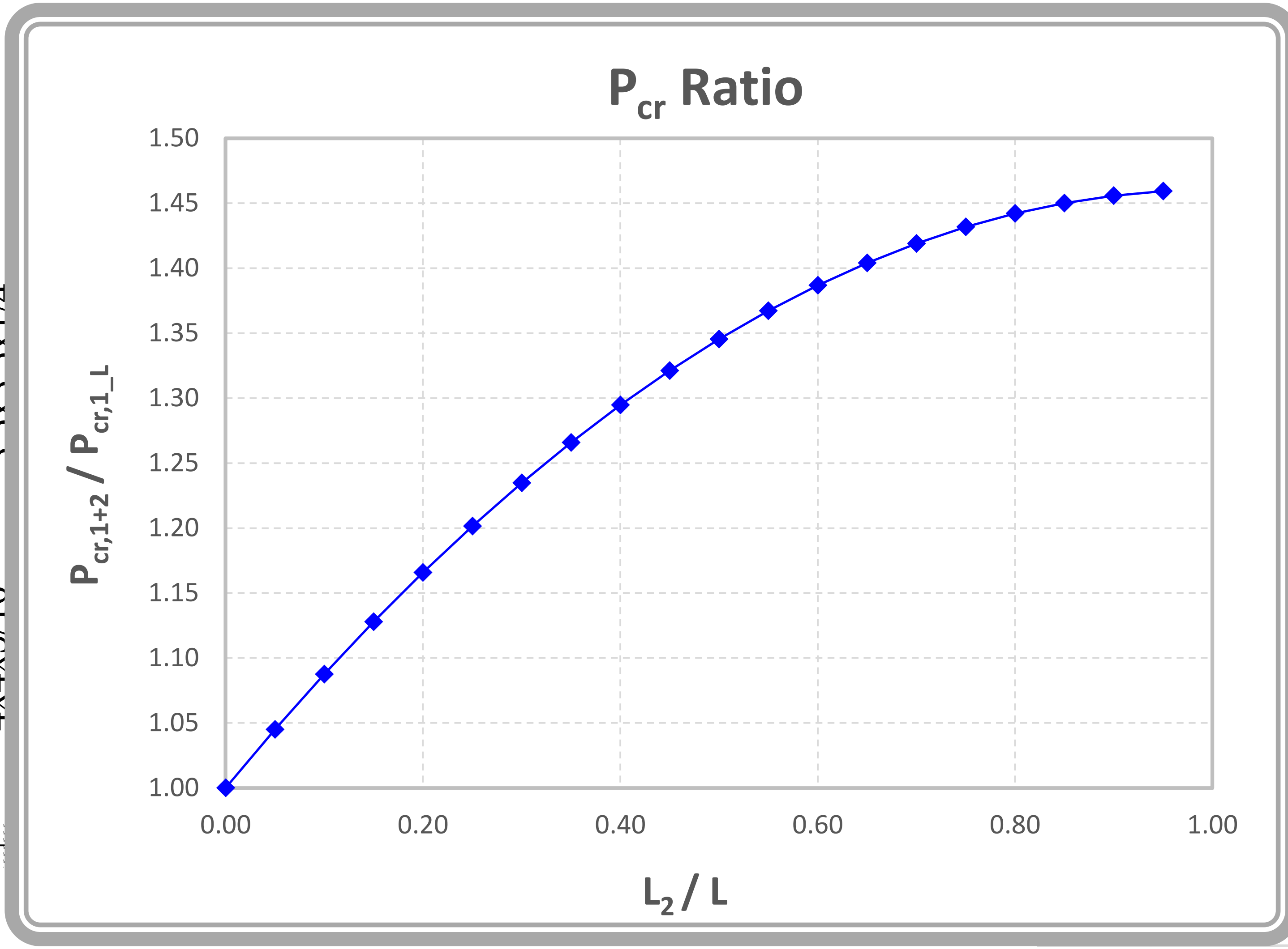
$$P_{cr} = \frac{\pi^2 EI_1}{l^2} \frac{1}{\left(\frac{l_1}{l} + \frac{l_2}{l}\frac{I_1}{I_2}\right) + \frac{1}{2\pi}\left(1 - \frac{I_1}{I_2}\right)\sin\frac{2\pi l_2}{l}}$$

IV. VALIDATION : Short

3.5x3.5x1/4

4x4x5/16

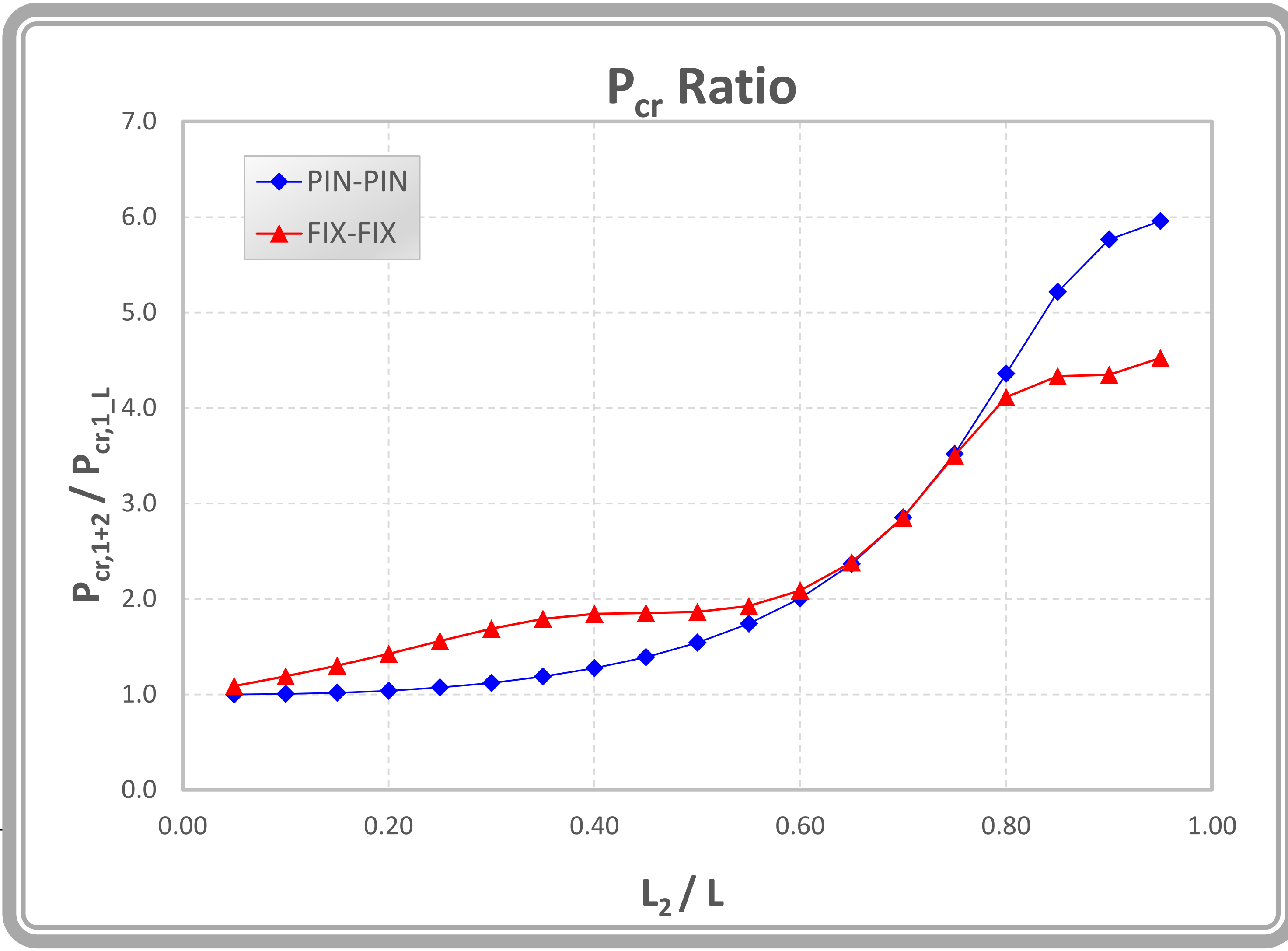
y



$P_{cr,1+2\_L}$
74.24

$P_{cr,1+2}/P_{cr,1\_L}$
1.04
1.09
1.13
1.17
1.20
1.23
1.27
1.29
1.32
1.35
1.37
1.39
1.40
1.42
1.43
1.44
1.45
1.46
1.46

IV. VALIDATION : Slender



Equivalent Unbraced Length Method

- Calculate the equivalent unbraced length ratio with respect to segment 1

$$RL_{equ} = L_1/L$$

- Use the same modeling methodology (model with a smaller section)

OPTION 1 : Equivalent Unbraced Length Method

- Calculate the equivalent unbraced length ratio with respect to segment 1

$$L_{e,F-F} = \sqrt{\frac{4\pi^2 EI_1}{P_{cr,1+2}}} \qquad L_{e,P-P} = \sqrt{\frac{\pi^2 EI_1}{P_{cr,1+2}}} \qquad RL_{equ} = L_e/L$$

- Use the same modeling methodology

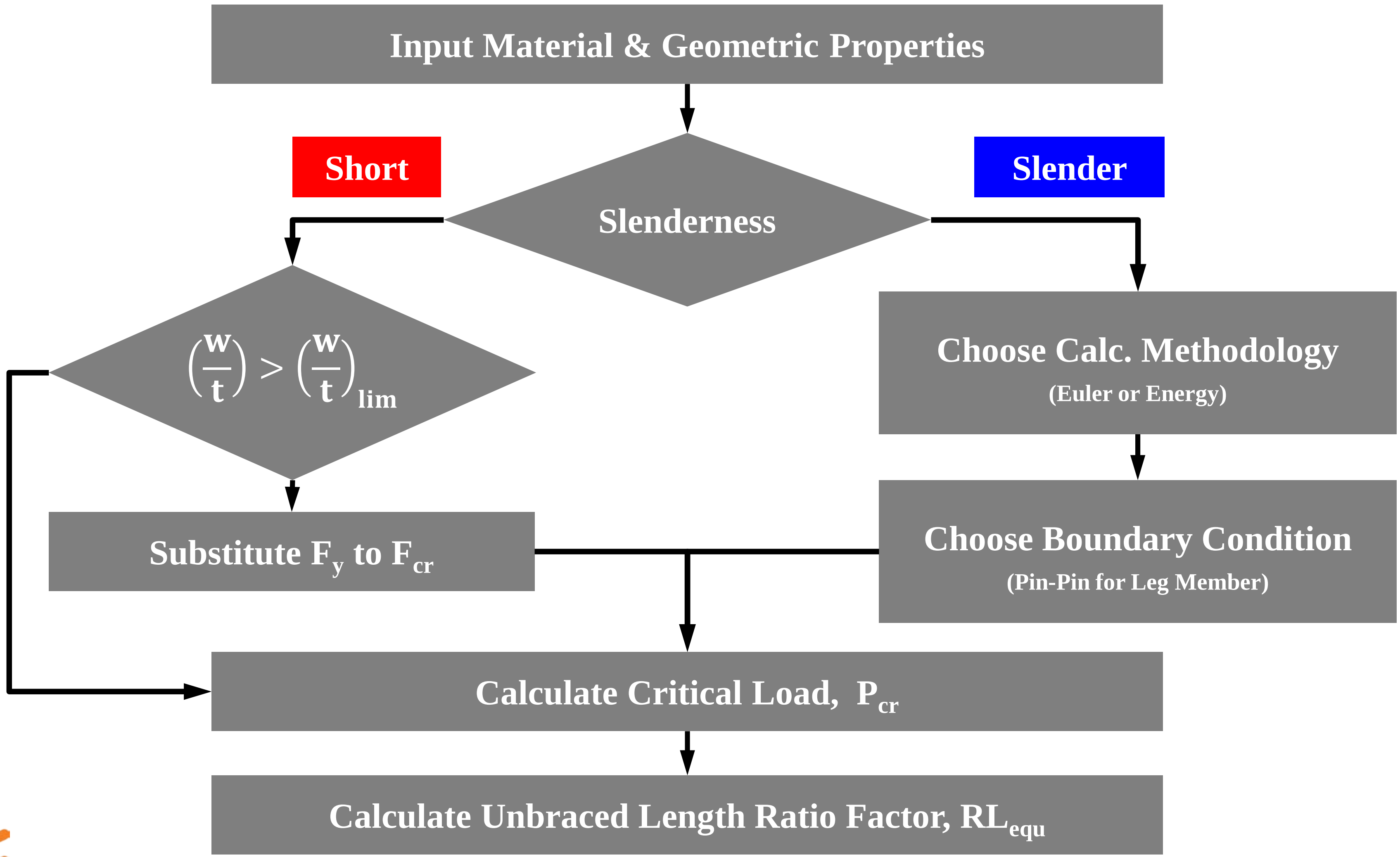
OPTION 2 : Equivalent Cross Sectional Method

- Calculate the moments of inertia satisfying P_{cr}

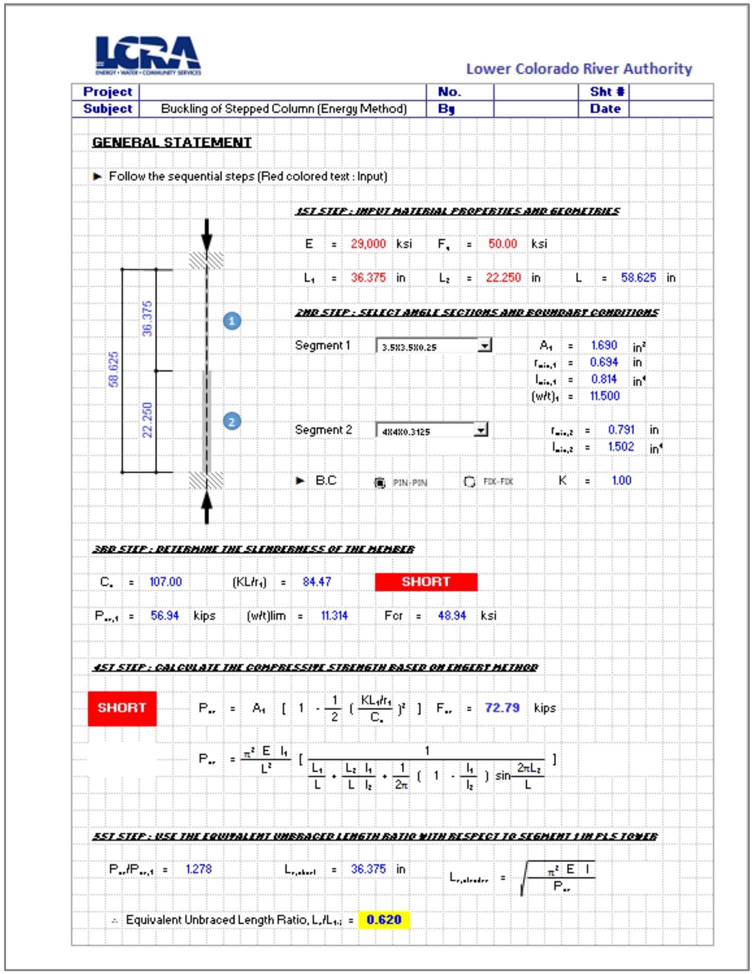
$$I_{equ,F-F} = \frac{P_{cr,1+2}L^2}{4\pi^2 E} \qquad I_{equ,P-P} = \frac{P_{cr,1+2}L^2}{\pi^2 E}$$

- Find the corresponding angle section and substitute

V. APPLICATION : Algorithm



2 SEGMENT – ENERGY METHOD


Lower Colorado River Authority

Project		No.		Sht #	
Subject	Buckling of Stepped Column (Energy Method)	By		Date	

GENERAL STATEMENT

► Follow the sequential steps (Red colored text : Input)

1ST STEP : INPUT MATERIAL PROPERTIES AND GEOMETRIES

E = 29,000 ksi F_y = 50.00 ksi

L_1 = 36.375 in L_2 = 22.250 in L = 58.625 in

2ND STEP : SELECT ANGLE SECTIONS AND BOUNDARY CONDITIONS

Segment 1: 3.5X3.5X0.25 A_1 = 1.690 in² $r_{min,1}$ = 0.694 in $I_{min,1}$ = 0.814 in⁴ $(w/t)_1$ = 11.500

Segment 2: 4X4X0.3125 $r_{min,2}$ = 0.791 in $I_{min,2}$ = 1.502 in⁴

► B.C PIN-PIN FIX-FIX K = 1.00

3RD STEP : DETERMINE THE SLENDERNESS OF THE MEMBER

C_c = 107.00 (KL/r_1) = 84.47 SHORT

$P_{cr,1}$ = 56.94 kips $(w/t)lim$ = 11.314 Fcr = 48.94 ksi

4ST STEP : CALCULATE THE COMPRESSIVE STRENGTH BASED ON ENERGY METHOD

SHORT $P_{cr} = A_1 [1 - \frac{1}{2}(\frac{KL_1/r_1}{C_c})^2] F_y$ = 72.79 kips

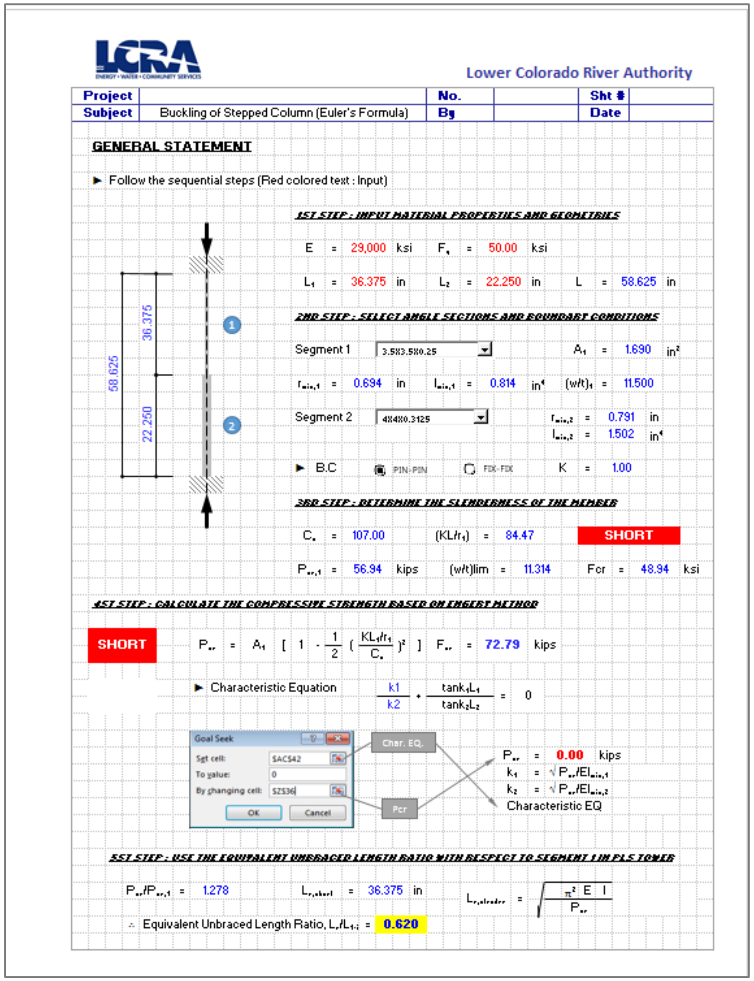
$P_{cr} = \frac{\pi^2 E I_1}{L^2} [\frac{1}{\frac{L_1}{L} + \frac{L_2}{L}\frac{I_1}{I_2} + \frac{1}{2\pi}(1 - \frac{I_1}{I_2})\sin\frac{2\pi L_2}{L}}]$

5ST STEP : USE THE EQUIVALENT UNBRACED LENGTH RATIO WITH RESPECT TO SEGMENT 1 IN PLS TOWER

$P_{cr}/P_{cr,1}$ = 1.278 $L_{r,short}$ = 36.375 in $L_{r,slender} = \sqrt{\frac{\pi^2 E I}{P_{cr}}}$

∴ Equivalent Unbraced Length Ratio, L_r/L_1 = 0.620

2 SEGMENT – EULER FORMULA


Lower Colorado River Authority

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Subject	Buckling of Stepped Column (Euler's Formula)	By		Date	

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SHORT $P_{cr} = A_1 [1 - \frac{1}{2}(\frac{KL_1/r_1}{C_c})^2] F_y$ = 72.79 kips

► Characteristic Equation $\frac{k1}{k2} + \frac{\tan k_1 L_1}{\tan k_2 L_2} = 0$

Goal Seek — Set cell: AC42 To value: 0 By changing cell: Z36 OK Cancel

Char. EQ. Pcr

P_{cr} = 0.00 kips $k_1 = \sqrt{P_{cr}/EI_{min,1}}$ $k_2 = \sqrt{P_{cr}/EI_{min,2}}$ Characteristic EQ

5ST STEP : USE THE EQUIVALENT UNBRACED LENGTH RATIO WITH RESPECT TO SEGMENT 1 IN PLS TOWER

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ANY QUESTIONS?