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DEVELOPMENT OF ANALYTICAL METHODS FOR SPLICED LEG MEMBERS

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ABSTRACT

This paper presents the development of analytical methods to estimate the compression capacity of a spliced leg member, a typical connection as a part of a latticed steel structure. Design and analysis of the spliced leg members typically assume the spliced leg member is a continuous member with a constant section instead of considering actual dimensions of two segments. Although, this approach has been accepted in the industry for a long time because it guarantees a conservative design, it has drawbacks like the overdesign of an individual member and an adverse effect to global stiffness. To address these problems, the rational analytical method is developed in this paper based on the Timoshenko's elastic stability theory and ASCE/SEI 10 standard. Timoshenko's theory was applied to slender members for the global buckling analysis using two theories, Euler Buckling and Energy Method, while the ASCE/SEI 10 standard was applied to short members for the local buckling analysis. In conclusion, this paper suggests a practical strategy to analyze spliced leg members in detail.

INTRODUCTION

In a lattice tower, we can't avoid the several connection joints along the main leg member causing a discontinuity of structural system, which is called "a spliced leg panel member". The spliced leg member can consist of either same angle section or different angle section. Regardless of combined angle sections, the spliced leg member should be considered as a discontinuous member due to the overlapped or plated connection portion. However, this spliced leg member has not been rationally analyzed and properly designed due to several reasons such as: 1) lack of understanding about the compression behavior of spliced leg member; 2) limitation of analysis/design toolS conventionally used in TL industry; 3) consideration of the spliced leg member as a less important member.

PLS tower program is a major FE program we are using every day to analyze the lattice tower. It is questionable that PLS tower can properly model and rationally analyze the spliced leg member. We usually model the spliced leg member as a continuous member with a constant section which is selected from the smaller section of two discontinuous members using a conventional unbraced length ratio. Figure 1 shows an example of modelling. As shown in the figure, neck member consisting of 3.5x3.5x0.25 and 4x4x0.1875 angle members was modeled with 3.5x3.5x0.25 member along the whole length. This approach is acceptable from the conservative design point of view as long as the member usage is under the unity. But, when it is over the unity, we need to justify or quantify the actual member capacity and to consider the consequences of this approach. Usually, we increase the size of the smaller angle section to avoid the failure resulting in an overdesign of the member. In addition, this approach can overestimate the overall structural behavior of the tower cause by the underestimation of stiffness matrix due to the modelling of smaller section. As shown in Figure 2, local stiffness between member 1-2 and member 2-3 has an overlapped portion which are k_{33}, k_{34}, k_{43}, k_{44}. It means that the local stiffness of the member can affect not only the behavior of the adjacent structural members but also the overall behavior of whole structure system.

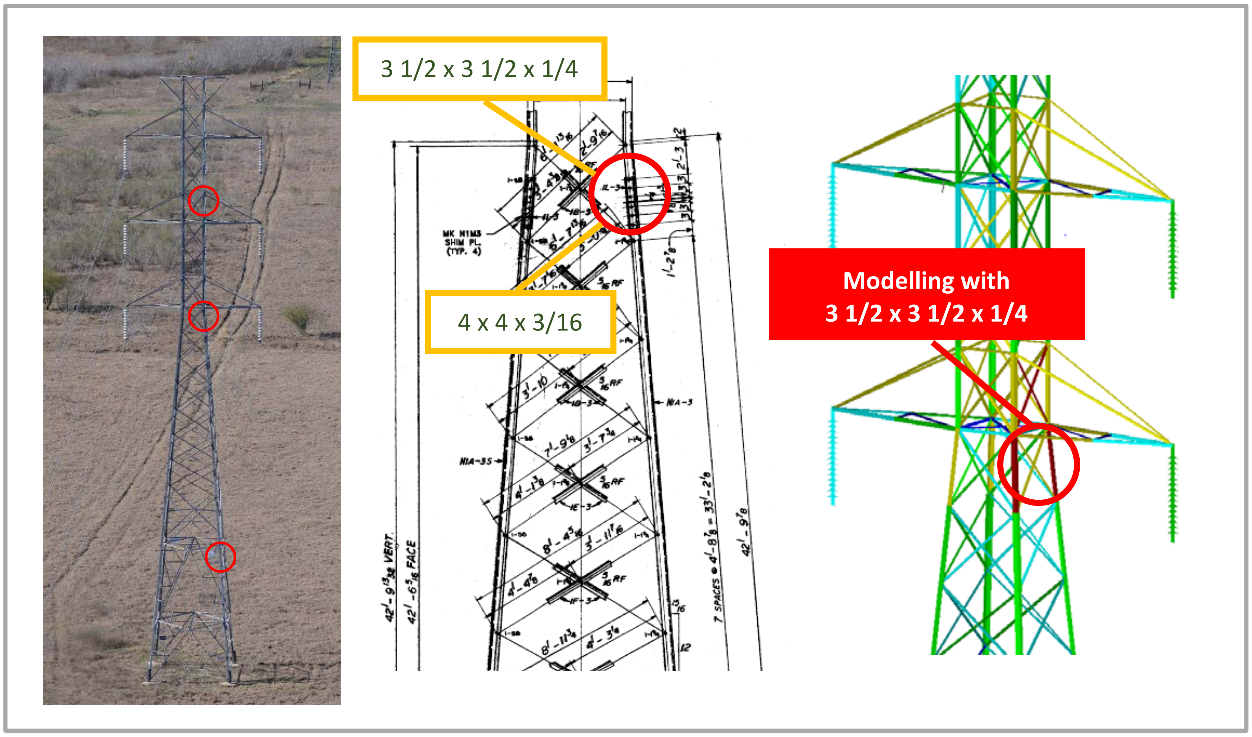


Figure 1 Spliced Leg Members in Lattice Tower

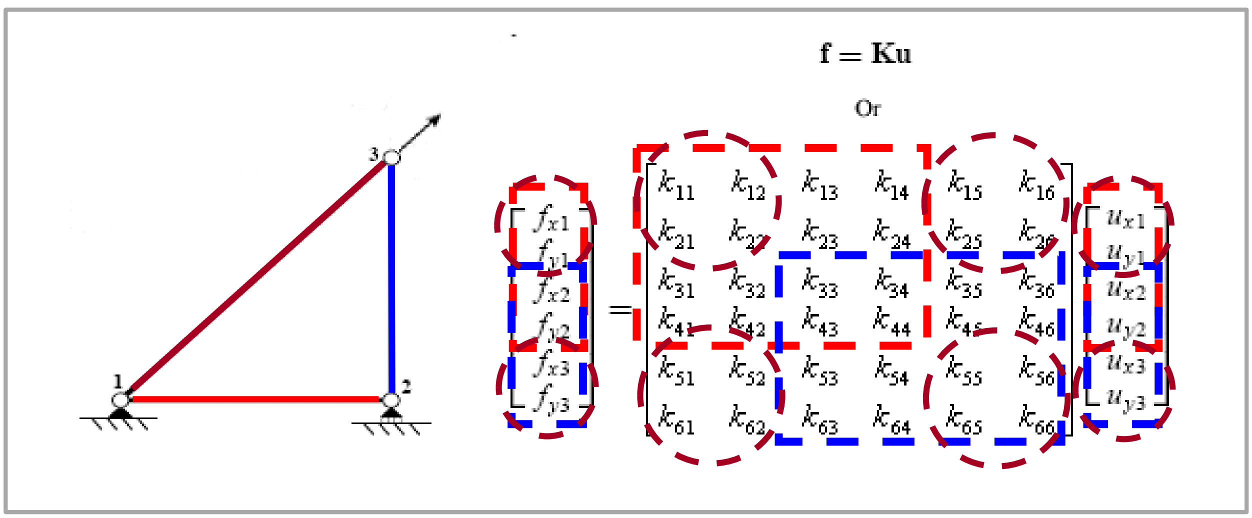


Figure 2 Spliced Leg Members in Lattice Tower

Even though it is not impossible to model two distinguished members in PLS tower program, it is still questionable how to assign the unbraced length ratio for each segment.

To address these problems, the rational analytical method is developed in this paper based on the Timoshenko's elastic stability theory and ASCE/SEI 10 standard and can be utilized in PLS tower program.

BACKGROUND THEORY

Compression Failure Modes of Thin Plate Structure

The failure modes of thin plate structures under the compression load can be categorized into the three modes which are local, distorsional, and overall buckling as shown in Figure 3. These failure modes are strongly affected by member slenderness ratio and fixity. Local and distorsional buckling failures are mostly observed in a short member which has a low slenderness ratio, while the overall buckling failure is observed in a long/slender member which has a high slenderness ratio

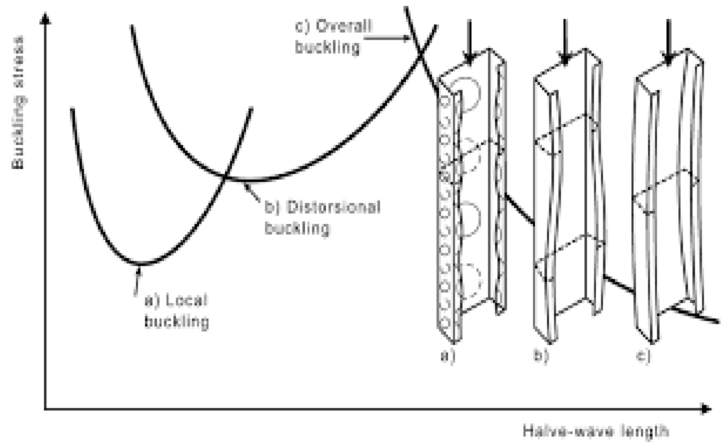


Figure 3 Compression Failure Modes

Short Member: ASCE 10 Code

ASCE 10 code provides the design compressive stresses for both short and slender angle members as shown below. The equation of the short member, Eq. (1), is an empirical formula, while the one of the slender member, Eq. (2), is theoretically derived based on Euler's Formula. This paper adopted the ASCE 10 code equation for the short member for developing the analytical method, but not for the slender member.

Design compressive stress of the short member is strongly affected by the w/t ratio (where w = flat width of angle, t = thickness of angle leg) as shown in Eqs. (3) to (5). The ranges of w/t ratio distinguish the compressive failure modes of the short member as we discussed earlier. Eqs. (3) to (5) represent a typical compression failure, local buckling failure, and distorsional failure, respectively.

$$\left[1-\frac{1}{2}\left(\frac{KL/r}{C_c}\right)^2\right] F_{cr} \qquad (KL/r \le C_c) \quad \text{for short member} \qquad (1)$$

$$\frac{\pi^2 E}{(KL/r)^2} \quad \left(r=\sqrt{I/A}\right) \qquad (KL/r > C_c) \quad \text{for slender member} \qquad (2)$$

where $C_c = \pi\sqrt{2E/F_y}$

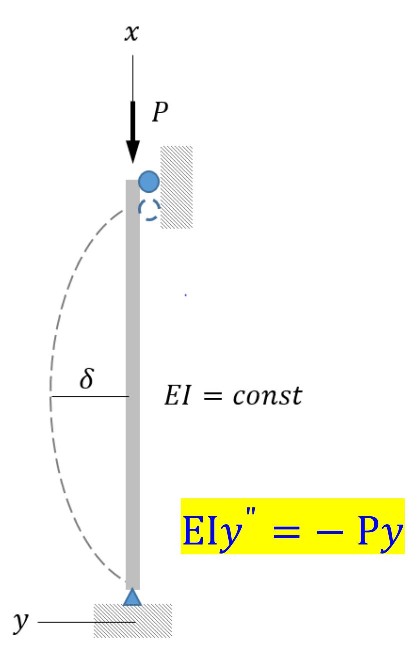
$$F_{cr} = F_y \qquad (\, w/t < 80/\sqrt{F_y}\,) \qquad (3)$$

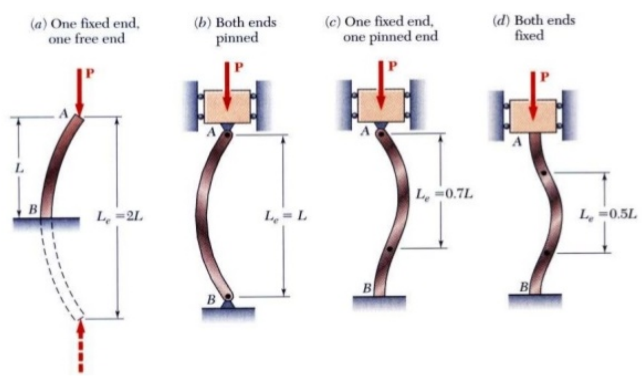
$$F_{cr} = \left[1.677 - 0.677\frac{w/t}{(w/t)_{lim}}\right] F_y \qquad (\, 80/\sqrt{F_y} \le w/t \le 144/\sqrt{F_y}\,) \qquad (4)$$

$$F_{cr} = \frac{0.0332\,\pi^2 E}{(w/t)^2} \qquad (\, 144/\sqrt{F_y} < w/t\,) \qquad (5)$$

Long/Slender Member: Euler's Formula

Two fundamental theories have been adopted in this paper to derive a characteristic equation for the analysis of a slender spliced leg member. The first theory is the Euler buckling formula, and the other is the energy method. The Euler formula is a basic theory to analyze the buckling behavior of the slender compressive member and would be familiar with most of structural engineer. We can get an exact solution by solving the second-order differential equation with respect to different boundary conditions as shown in Figure 4. From this theory, we can get a critical buckling load as well as calculate a deflection curve. It should be noted that the following differential equation is only for the slender member having a constant cross section along the member length. As shown in the figure, the critical buckling loads are varied with respect to different boundary conditions. Comparing the critical load of Pin-Pin B.C to the others with different B.C.s, the ratios of the critical loads of Fixed-Free, Fixed-Fixed, and Fixed-Pin boundary conditions are 0.25, 4, and 2.046, respectively. Similar to the buckling load comparison, effective length ratios (K, also called unbraced length ratio) are 2, 0.5, and 0.699 with respect to each boundary condition. We can observe that major important factors affecting the critical buckling loads are the boundary conditions and cross sectional property.





B.C	P_{cr}	L_e (=KL)
Pin-Pin	$\pi^2 EI/L^2$	L
Fix-Free	$\pi^2 EI/4L^2$	2L
Fix-Fix	$4\,\pi^2 EI/L^2$	0.5L
Fix-Pin	$2.046\,\pi^2 EI/L^2$	0.699L

Figure 4 Euler Buckling Loads under Different Boundary Conditions (B.C.)

Long/Slender Member: Energy Method

The main reason to use the energy method in problems of buckling is the determination of approximate values of critical loads for cases in which an exact solution of the differential equation of the deflection curve is either unknown or too complicated. In such cases we must begin by assuming a reasonable shape for the deflection curve. While it is not essential for an approximate solution that the assumed curve satisfy completely the end conditions of the compression member, it should at least satisfy the conditions pertaining to deflections and slopes. The critical buckling load can be obtained by solving the equilibrium equation, Eq. (6), between the strain energy of bending (ΔU) and the work done by the compressive force, P (ΔT). Table 1 summarizes the formulas of assumed deflection curves according to different boundary conditions.

$$\Delta U = \Delta T \tag{6}$$

$$\Delta U = \int_0^l \frac{M^2}{2EI}\,dx \qquad (M = Py) \tag{7}$$

$$\Delta T = P\lambda \qquad \left(\lambda = \frac{1}{2}\int_0^l \left(\frac{dy}{dx}\right)^2 dx\right) \tag{8}$$

Table 1 Assumed Deflection Curve

B.C	DEFLECTION, y
Pin-Pin	$\sin(\pi x/l)$
Fix-Free	$\delta\left(1 - \cos\frac{\pi x}{2l}\right)$
Fix-Fix	$\frac{\delta}{2}\left(1 - \cos\frac{2\pi x}{l}\right)$

GOVERNING EQUATIONS

Short Member: ASCE 10 Code

3D nonlinear FE analysis has been conducted to investigate the compressive behavior of short spliced leg member consisting of two different angle sections using ANSYS. Analysis was carried out by differentiating the length of section 1 member, L_1, with respect to whole member length, L, like 0.8L, 0.6L, 0.4L, and 0.2L. Analysis results show that section 1 member governs the final failure of short spliced member regardless of the considered length ratios as shown in Figure 5. Local and distorsional buckling failure modes can be observed according to different length ratios. Final failure loads of each case tend to approach the failure load of section 1 member with its length, L_1, calculated by ASCE 10 code equation. Accordingly, following equation, Eq. (9), is suggested to calculate the critical buckling load of the short spliced leg member.

$$P_{cr} = A_1\left[1 - \frac{1}{2}\left(\frac{KL/r}{C_c}\right)^2\right] F_{cr} \qquad (K = L_1/L) \tag{9}$$

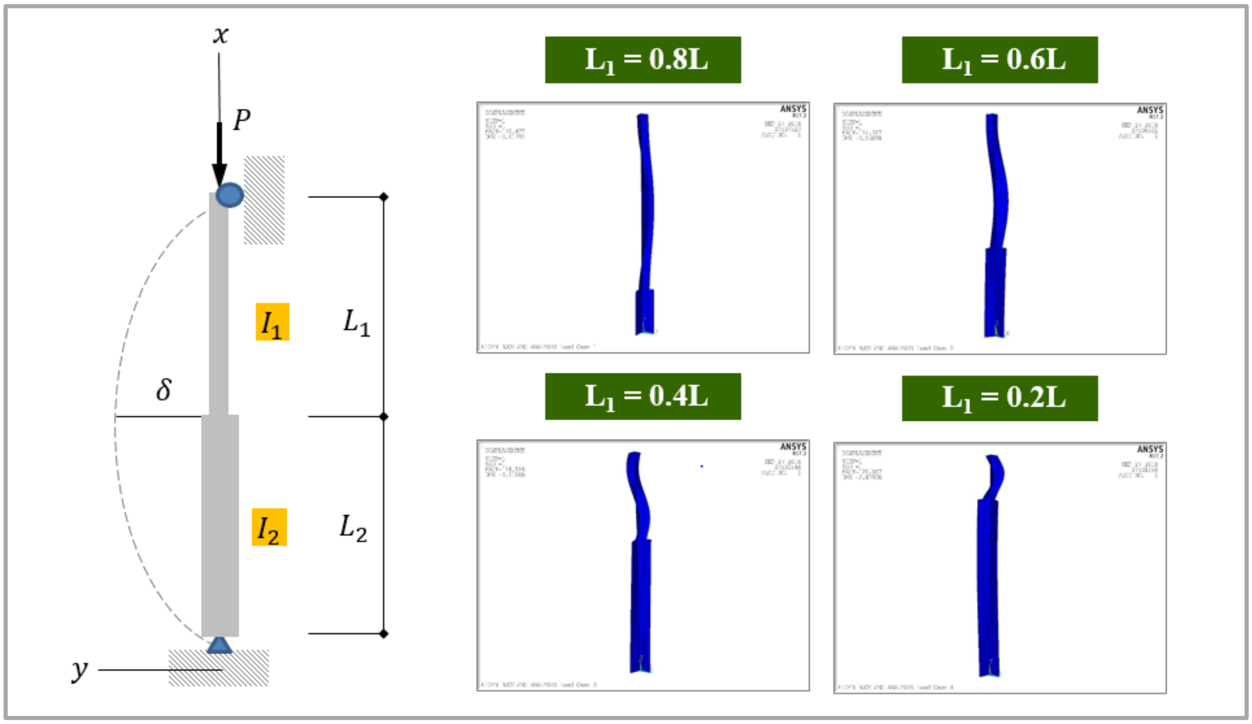


Figure 5 3D Nonlinear Analysis Results for the Short Member

Long/Slender Member

Governing equations are derived with respect to three configurations as shown in Figure 6 which are two segments with pin-pin boundary condition, two segments with fix-fix boundary condition, and three segments with pin-pin boundary condition.

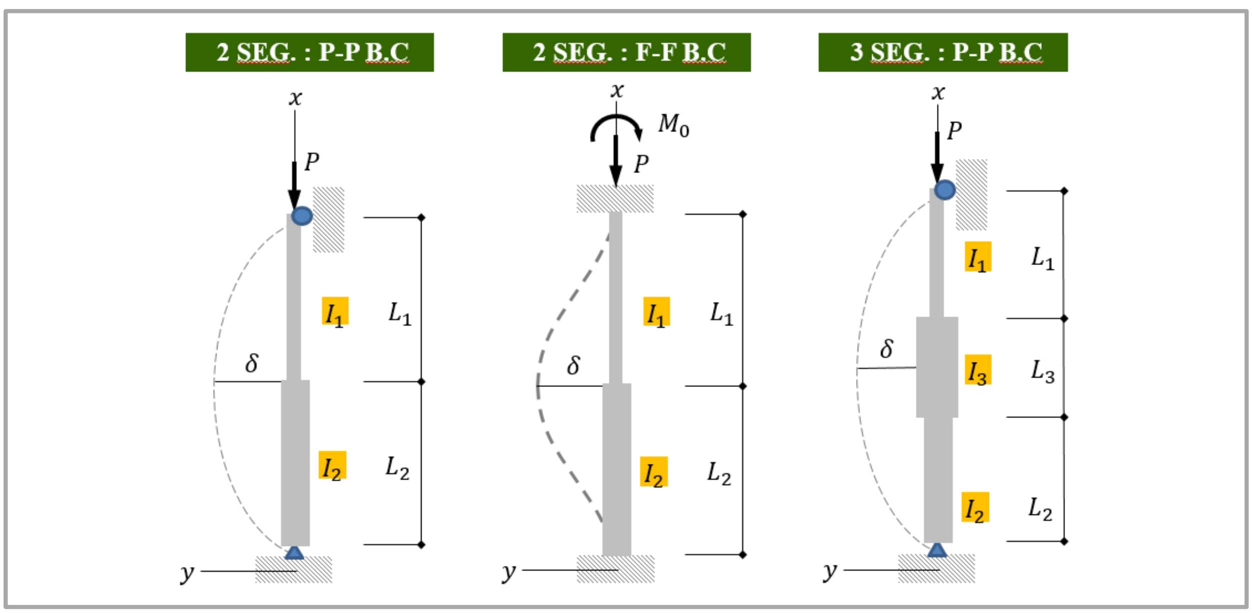


Figure 6 Three Long/Slender Member Configurations

Euler's Formula

Table 2 summarizes the Euler's Formula for each configuration including second-order differential equations, solutions, boundary conditions, and initial conditions.

Table 2 Euler Formulas for Three Configurations

CASE	DIFF. EQ	SOLUTION	B.C & I.C
2 SEGs P-P B.C.	$EI_1 y_1'' = -Py_1$ $EI_2 y_2'' = -Py_2$ $y_1'' + k_1^2 y_1 = 0$ $y_2'' + k_2^2 y_2 = 0$ $\begin{pmatrix} k_1^2 = P/EI_1 \\ k_2^2 = P/EI_2 \end{pmatrix}$	$y_1 = A\cos k_1 x + B\sin k_1 x$ $y_2 = C\cos k_2 x + D\sin k_2 x$	$y_2(0) = y_1(L) = 0$ $y_2(L_2) = y_1(L_2) = \delta$
2 SEGs F-F B.C.	$EI_1 y_1'' = -Py_1 + M_0$ $EI_2 y_2'' = -Py_2 + M_0$ $y_1'' + k_1^2 y_1 = M_0/EI_1$ $y_2'' + k_2^2 y_2 = M_0/EI_2$ $\begin{pmatrix} k_1^2 = P/EI_1 \\ k_2^2 = P/EI_2 \end{pmatrix}$	$y_1 = A\cos k_1 x + B\sin k_1 x + M_0/P$ $y_2 = C\cos k_2 x + D\sin k_2 x + M_0/P$	$y_2(0) = y_1(L) = 0$ $y_2'(0) = y_1'(L) = 0$ $y_2(L_2) = y_1(L_2) = \delta$ $y_2'(L_2) = y_1'(L_2) = 0$
3 SEGs P-P B.C.	$EI_1 y_1'' = -Py_1$ $EI_2 y_2'' = -Py_2$ $EI_3 y_3'' = -Py_3$ $y_1'' + k_1^2 y_1 = 0$ $y_2'' + k_2^2 y_2 = 0$ $y_3'' + k_3^2 y_3 = 0$ $\begin{pmatrix} k_1^2 = P/EI_1 \\ k_2^2 = P/EI_2 \\ k_3^2 = P/EI_3 \end{pmatrix}$	$y_1 = A\cos k_1 x + B\sin k_1 x$ $y_2 = C\cos k_2 x + D\sin k_2 x$ $y_3 = E\cos k_3 x + F\sin k_3 x$	$y_2(0) = y_1(L) = 0$ $y_2(L_2) = y_3(L_2) = \delta_2$ $y_2'(L_2) = y_3'(L_2)$ $y_3'(L_2 + L_3) = y_1'(L_2 + L_3)$ $y_3(L_2 + L_3) = y_1(L_2 + L_3) = \delta_1$

By solving the above differential equations, we can get the governing equations for each configuration except for the case of 3 segments with pin-pin boundary condition. This case is too complicated to solve. Critical buckling load of this case will be obtained through the energy method. Eq. (10) is a governing equations for the case of 2 segments with pin-pin boundary condition, while Eq. (11) is for 2 segments with fix-fix boundary condition

$$\frac{k_1}{k_2} + \frac{\tan k_1 L_1}{\tan k_2 L_2} = 0 \tag{10}$$

$$\frac{k_2}{k_1} + \frac{\tan k_1 L_1}{\tan k_2 L_2} = 0 \tag{11}$$

Energy Method

Table 3 summarizes the equations used for the energy method according to Eq. (6). Approximate critical buckling loads for each case can be obtained by solving the equilibrium equations as show in Eqs. (12), (13), and (14).

Table 3 Euler Formulas for Three Configurations

CASE	DEF. CURVE, y	STRAIN ENERGY, ΔU	WORK, ΔT
2 SEGs P-P B.C.	$\delta\sin(\pi x/l)$	$\int_0^{l_2}\frac{M^2}{2EI_2}dx + \int_{l_2}^{l}\frac{M^2}{2EI_1}dx$	$\frac{P}{2}\int_0^l\left(\frac{dy}{dx}\right)^2dx$
2 SEGs F-F B.C.	$\frac{\delta}{2}\left(1-\cos\frac{2\pi x}{l}\right)$	$\int_0^{l_2}\frac{M^2}{2EI_2}dx + \int_{l_2}^{l}\frac{M^2}{2EI_1}dx$	$\frac{P}{2}\int_0^l\left(\frac{dy}{dx}\right)^2dx$
3 SEGs P-P B.C.	$y = \delta\sin(\pi x/l)$	$\int_0^{l_2}\frac{M^2}{2EI_2}dx + \int_{l_2}^{l_2+l_3}\frac{M^2}{2EI_3}dx + \int_{l_2+l_3}^{l}\frac{M^2}{2EI_2}dx$	$\frac{P}{2}\int_0^l\left(\frac{dy}{dx}\right)^2dx$

$$P_{cr} = \frac{\pi^2 EI_1}{l^2}\frac{1}{\left(\frac{l_1}{l}+\frac{l_2}{l}\frac{I_1}{I_2}\right)+\frac{1}{2\pi}\left(1-\frac{I_1}{I_2}\right)\sin\frac{2\pi l_2}{l}} \tag{12}$$

$$P_{cr} = \frac{4\pi^2 EI_1}{l^2}\frac{1}{3\left(\frac{l_1}{l}+\frac{l_2}{l}\frac{I_1}{I_2}\right)+\frac{1}{4\pi}\left(1-\frac{I_1}{I_2}\right)\left(8\sin\frac{2\pi l_2}{l}-\sin\frac{4\pi l_2}{l}\right)} \tag{13}$$

$$P_{cr} = \frac{\pi^2 EI_1}{l^2}\frac{1}{\left(\frac{l_1}{l}+\frac{l_3}{l}\frac{I_1}{I_3}+\frac{l_2}{l}\frac{I_1}{I_2}\right)+\frac{1}{2\pi}\left\{\left(\frac{I_1}{I_3}-1\right)\sin\frac{2\pi l_1}{l}+\left(\frac{I_1}{I_3}-\frac{I_1}{I_2}\right)\sin\frac{2\pi l_2}{l}\right\}} \tag{14}$$

VALIDATION

Developed methods are validated against the real short and slender structural members used in an existing lattice tower.

Short Member

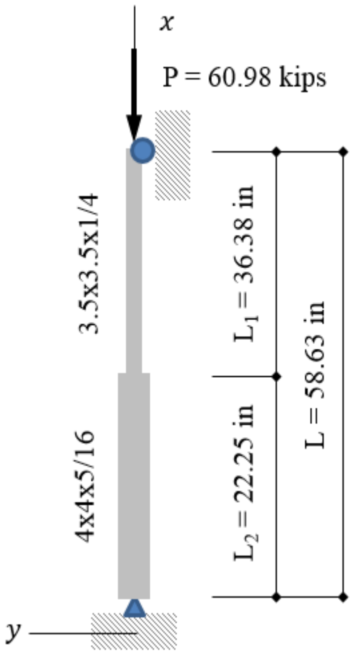


Figure 7 Short Member Example

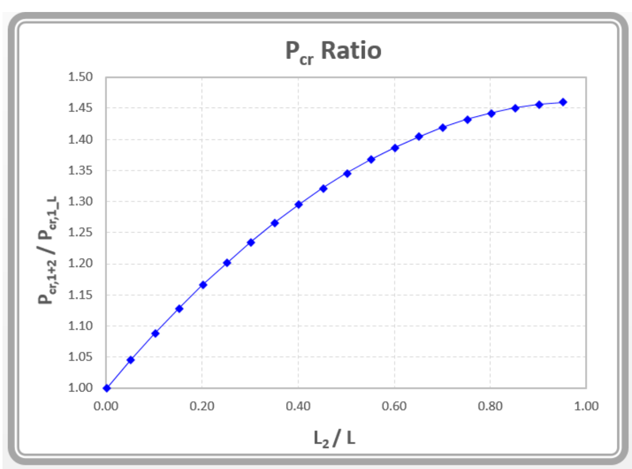
Figure 7 shows the configuration of the short member used for the validation. Angle size of smaller section of the spliced leg member is 3.5x3.5x1/4 with 36.38 in length, while larger one is 4x4x5/16 with 22.25 in length.

When this member is considered as a continuous member assuming the constant section having a smaller and larger section, critical buckling loads of each case are 57.86 kips and 91.14 kips, respectively. As mentioned earlier, for the sake of conservative design, 57.86 kips is considered as a design compression capacity in practical design. This approach can result in a compression failure as UC value with 1.05. While the critical buckling load of the spliced leg member calculated by the developed method is 74.24 kips. This value reduce the UC from 1.05 to 0.82. It can be observed from this example that a rational/practical analysis approach can avoid an unnecessary overdesign.

$$UC = \frac{F_{tower}}{P_{cr,1\_L}} = \frac{60.98}{57.86} = 1.05 \quad (14)$$

$$UC = \frac{F_{tower}}{P_{cr,1+2\_L}} = \frac{60.98}{74.24} = 0.82 \quad (15)$$

Effect of segment length ratio to the critical buckling load is examined by varying the each segment length. Figure 8 shows that the buckling loads of spliced member proportionally increase at the certain range of L_2/L, then start to gradually increase after 0.6. This tendency can explain that the influence of larger segment stiffness is diminished at a certain level as the portion of the segment is getting bigger.



L_2 / L	$P_{cr,1+2}$	$P_{cr,1+2}/P_{cr,1\_L}$
0.05	59.78	1.04
0.10	62.22	1.09
0.15	64.52	1.13
0.20	66.70	1.17
0.25	68.74	1.20
0.30	70.65	1.23
0.35	72.43	1.27
0.40	74.07	1.29
0.45	75.59	1.32
0.50	76.97	1.35
0.55	78.22	1.37
0.60	79.34	1.39
0.65	80.33	1.40
0.70	81.19	1.42
0.75	81.91	1.43
0.80	82.50	1.44
0.85	82.97	1.45
0.90	83.29	1.46
0.95	83.49	1.46

Figure 8 Critical Buckling Loads of Short Member According to Segment Length Ratio

Short Member

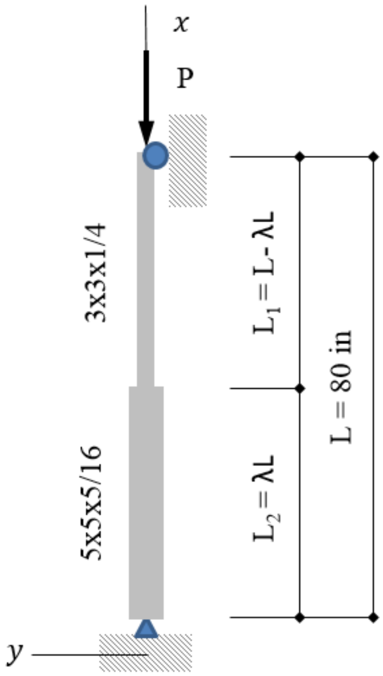


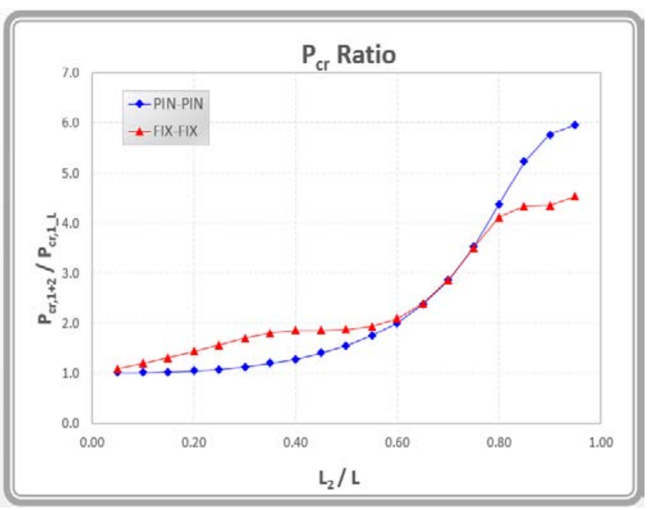
Figure 9 Slender Member Example

Figure 9 shows the configuration of the slender member used for the validation. Angle size of smaller section of the spliced leg member is 3x3x1/4, while larger one is 5x5x5/16. Total length of the spliced member is assumed as 80 in and the segment length ratios are varied. Validation was conducted with respect to pin-pin and fixed-fixed boundary conditions.

Similar to the example of short member, buckling loads of continuous members with a smaller and larger section are 55.45 kips and 221.82 kips under the pin-pin boundary condition, while they are 221.82 kips and 1327.34 kips under the fix-fix boundary condition.

It is observed that the buckling loads of the spliced member are converging to the buckling loads of constant section members regardless of boundary conditions as shown in the table of Figure 10. This observation can validate the developed method in this paper.

Unlike the short member, the buckling load ratio is exponentially increased as the L_2/L ratio increase. From this observation, it can be concluded that the influence of larger segment stiffness is not significant when its portion over the spliced member is small.



L_2 / L	PIN-PIN		FIX-FIX	
	$P_{cr,1+2}$	$P_{cr,1+2}/P_{cr,1\_L}$	$P_{cr,1+2}$	$P_{cr,1+2}/P_{cr,1\_L}$
0.05	55.49	1.001	241.50	1.089
0.10	55.75	1.005	263.73	1.189
0.15	56.40	1.017	288.72	1.302
0.20	57.60	1.039	316.36	1.426
0.25	59.48	1.073	345.77	1.559
0.30	62.17	1.121	374.54	1.688
0.35	65.84	1.187	397.58	1.792
0.40	70.71	1.275	409.23	1.845
0.45	77.12	1.391	411.21	1.854
0.50	85.52	1.542	413.36	1.863
0.55	96.56	1.741	427.29	1.926
0.60	111.28	2.007	462.82	2.086
0.65	131.14	2.365	528.59	2.383
0.70	158.25	2.854	633.04	2.854
0.75	195.05	3.517	776.99	3.503
0.80	241.81	4.360	912.25	4.113
0.85	289.22	5.215	961.41	4.334
0.90	319.59	5.763	964.68	4.349
0.95	330.43	5.959	1003.79	4.525

Figure 10 Critical Buckling Loads of Slender Member According to Segment Length Ratio

APPLICATION IN PLS TOWER

This paper suggests the practical methodologies to apply the developed analysis methods to PLS tower modeling and analysis.

Short Member

For a short spliced member, it is suggested to follow the conventional modelling technique, that is, to model the spliced member as a constant section member along the whole length assuming the smaller section. By applying an equivalent unbraced length ratio shown below, it is possible to correctly analyze the short spliced member.

$$RL_{equ} = L_1/L \tag{16}$$

Slender Member

There are two methods to incorporate the developed analysis to the slender spliced member. The first method is similar to the method applied to the short spliced member. The spliced member is modeled with a constant section of smaller section and analyzed with an equivalent unbraced length ratio which is developed in this paper.

$$L_{e,F-F} = \sqrt{\frac{4\pi^2 EI_1}{P_{cr,1+2}}} \qquad L_{e,P-P} = \sqrt{\frac{\pi^2 EI_1}{P_{cr,1+2}}} \qquad RL_{equ} = L_e/L \tag{17}$$

The second method is to substitute the spliced member with an equivalent sectional member as shown below. It can be called as an equivalent cross sectional method.

$$I_{equ,F-F} = \frac{P_{cr,1+2}L^2}{4\pi^2 E} \qquad I_{equ,P-P} = \frac{P_{cr,1+2}L^2}{\pi^2 E} \tag{18}$$

Both methods have strengths and weaknesses. The equivalent unbraced length method is familiar to us and easy. However, it can’t capture the actual structural behavior of the spliced member compared to the equivalent cross sectional method because this method can’t represent the stiffness of the spliced member. The equivalent cross sectional method has an advantage to capture the structural behavior, but it is hard to find the exactly matched equivalent angle section in angle library.

To conclude, the equivalent unbraced length method is more practical and easy to apply.

Algorithm and Calculation Sheet

Calculation sheet is developed to use the developed analysis method. Figure 11 shows the algorithm adopted in the developed calculation sheet, while Figure 12 shows the developed calculation sheet.

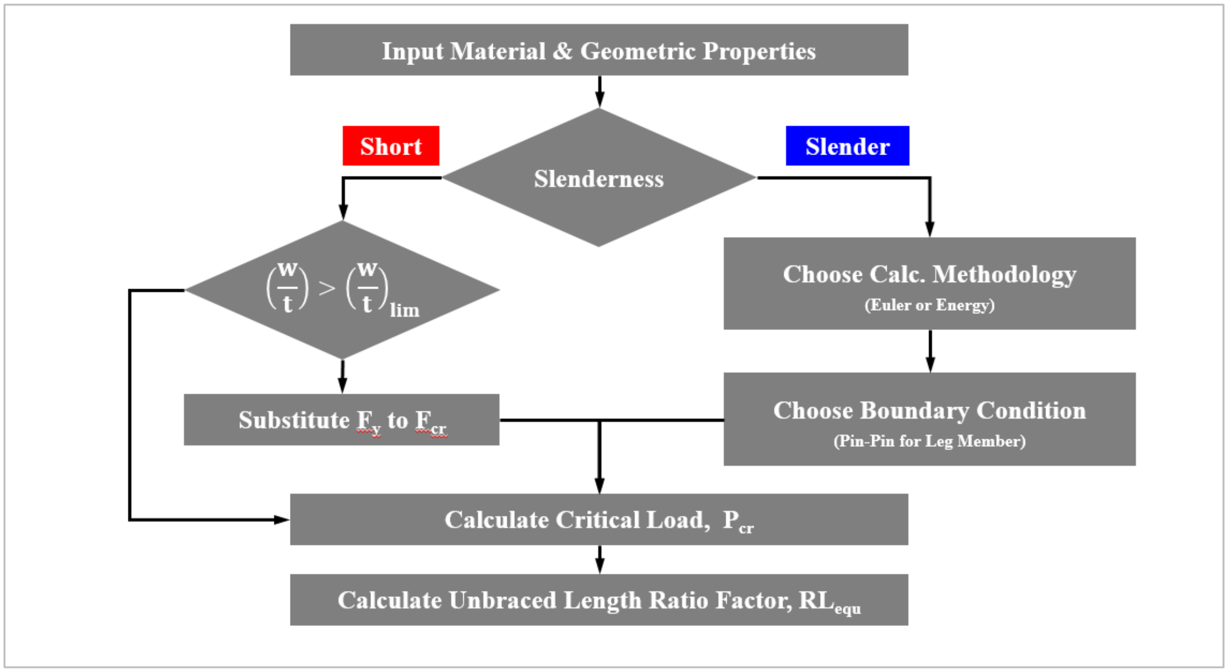


Figure 11 Algorithm Chart of Developed Analysis Method

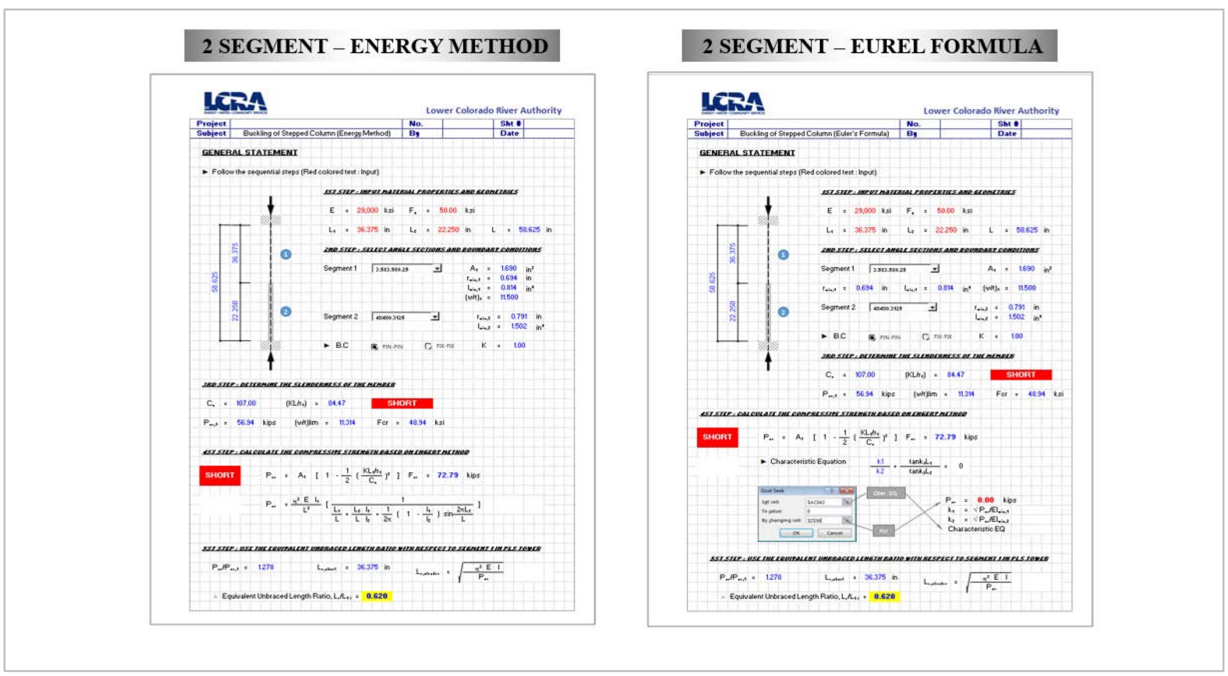


Figure 12 Developed Calculation Sheet

CONCLUSION

This paper presents the development of analysis methods for a spliced leg member. The philosophy of ASCE 10 code was adopted for the short spliced leg member, while Timoshenko's theory of elastic stability was applied to the buckling analysis of the slender spliced leg member. Developed methods were validated against the existing structure members. Finally, this paper suggests a rational/practical strategy to analyze spliced leg members.

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