Adaptive and Robust Harmonic Estimation and Mitigation

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Abstract

Harmonic distortion affects the quality of control of power grid, and estimation of harmonic distortion is an important task for controlling of power systems. Harmonics are caused by nonlinear loads such as power inverters, variable speed drives, and other inductive loads. Harmonics decrease the quality of power and cause problems for power systems such as equipment damage, high loading, inaccurate metering, outages, and system losses. As the level of technology increases, more and more nonlinear devices and load are being served by power systems and its becoming increasingly important to reduce the harmonics created by these devices. There are many different techniques for mitigating harmonic components such as passive harmonic filters, multi-pulse rectifiers, active harmonic filters, and hybrid harmonic filters. However, some of these techniques require extensive analysis in order for engineers to identify the harmonic signals and design the filters to properly mitigate them. This paper will be proposing a couple of methods to analyze and estimate the harmonic components of a given signal. The algorithms will be based on principles of adaptive control and parameter estimation and they will allow for accurate signal estimation with little error. The algorithms are verified via simulations with generated sample signals and signals from a real voltage inverter. The algorithms have been proven to significantly simplify the modeling and simulation of converters.

This paper aims to show that it is possible to perform analysis on harmonic distortions and identify the harmonic components of a signal. With a sample output signal, one can correctly estimate the harmonic contents of a system quickly, accurately, and with minimal prior knowledge of the system. With the harmonic estimates, one can then design the filters that will be used for mitigation.

Introduction

Harmonics are defined as signal components contained in integer multiples of the fundamental frequency [1]. These higher order frequency components then cause disturbances to the voltage or current signals which distorts the fundamental voltage and current signal. The result is a signal that is not a pure sinusoid, and this can cause problems both for the supplier and consumer. This leads to the question, what is the cause of harmonics and why do they exist in power systems?

Harmonics are caused by non-linear loads that exist on the system. In a standard case with a linear load, the voltage and current signals will exist as pure sinusoids and will be proportional to each other in relation to the size of the load that is being supplied. This is because in a linear case the load is constant and the impedance is not affected by the frequency. An example can be seen in the figure below.

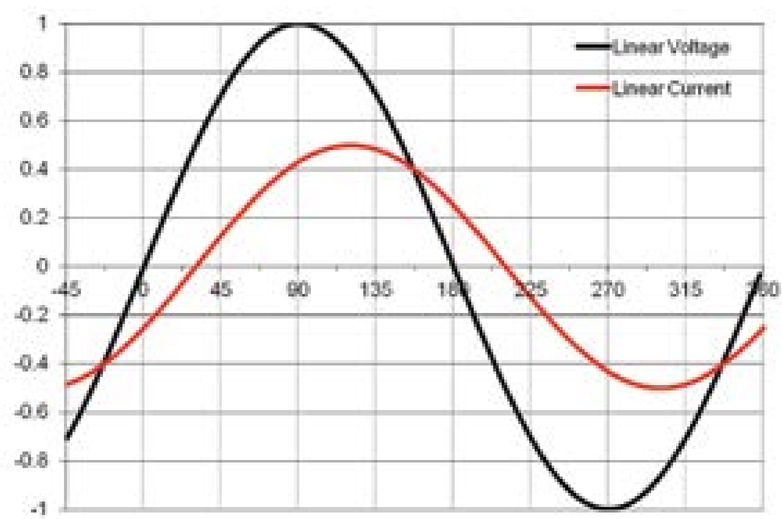

Figure 1: Linear Voltage and Current

Conversely, a non-linear load behaves in a very different matter when compared the linear load. There are a few things that make a load non-linear. The use of components such as diodes and thyristors, which are normally found in power converters and inverters, do not allow for normal current flow and cause the applied source voltage to be non-linear. Another characteristic of a non-linear load is that the impedance of the load is affected by frequency. Since the impedance is constantly changing due to the frequency then the current suppling the load will be will be non-linear, and this will in turn influence the voltage source serving the load. The non-linearities will distort the voltage and have adverse effects on the entire system. An example of a non-linear system is shown below.

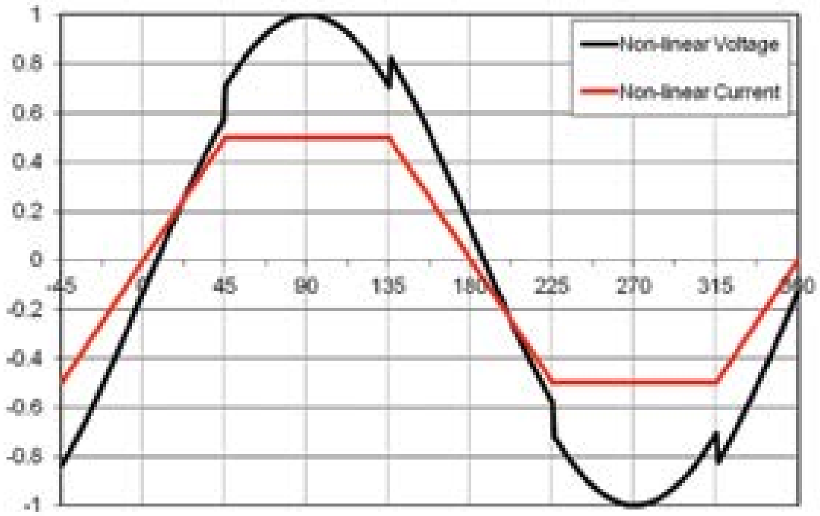

Figure 2: Non-linear Voltage and Current

Problems Caused by Harmonics

These harmonic distortions cause a variety of problems on a power system. These effects include things such as overloads in the neutral, unexpected power supply failures, reduced efficiency and lifetime on connected loads, damage to sensitive equipment, system losses, communication interferences, low voltage, and high equipment loading. These side effects stress the system in unnecessary ways and increase costs to the power supplier [2].

One example is unexpected power failures. These are caused by reclosers and fuses operating by seeing increased fault currents due to harmonics. The nonlinear currents are much higher than linear currents from the same power level, which may cause a device to operate much sooner than expected. Outages and system losses incur great cost to the supplier due to loss of revenue and associated costs of service restoration. Loss of service effects outage indexes such as SAIDI, CAIDI, SAIFI, and ASAI. Power suppliers are required to meet certain service standards and harmonic disturbances have potential to reduce those services at cost to the power supplier [1].

The nonlinear currents caused by harmonics are also responsible for higher than normal loading on equipment which can cause damage to the equipment. Transformers will have increased losses due to heating and higher eddy currents and hysteresis. Harmonics will also cause excessive heat in induction motors which will cause damage, reduce motor life, and increase losses. Finally, there will be increased losses in cables as the resistance of the current is dependent on the frequency. The higher frequency harmonics will increase the resistance and heat up the conductor causing heat losses. These factors lead to increased costs for the both the power supplier and the consumer as equipment will either need to be replaced or oversized to handle the harmonic loading.

Another costly area that harmonics will affect is metering. Some metering devices are not able to read the harmonic currents. This is because during phasor analysis the harmonic frequencies are rotating at a faster rate compared to the fundamental, and meters are not able to see the affects that the harmonics have on the fundamental RMS current. When a consumers' load is producing a lot of harmonics, they are drawing more power than what the meter can read due to the higher frequency components of the harmonics. In some cases, it's possible that the neutral current can draw up to 30% more current than the meter is actually reading, and this results in monetary losses for the power supplier [3].

Importance of Harmonic Estimation

Harmonics cause many system problems such as the ones stated above, and many more. The importance of good power quality is becoming even greater in modern times as there are more loads coming online that introduce harmonics to systems. Improving and maintaining high power quality is one of the main standards that system planning engineers must consider for system planning studies. This can be difficult because loads are constantly fluctuating, and the harmonics vary with load. Usually power quality will be improved based on readings at peak demand times. This is effective, however when the system is not experiencing peak demand, the power quality improvements and harmonic compensation will exceed the needs of the system and will introduce leading power factor which is also not ideal for the system.

With the implementation of harmonic parameter estimation, power supplier companies and system planning engineers can better prepare for both peak and non-peak demand times for their systems. With parameter estimation, harmonic mitigation and power quality improvements can be done with higher accuracy and with more adaptability than previously possible. This means that problems such as high equipment loading, inaccurate metering, equipment failures, system outages, and losses will be significantly reduced, and general system reliability will increase.

Methods

Problem Formulation

An expression must first be found to express the harmonics in the system. It was determined to follow a Fourier series format so that each of the harmonic components could be identified. The following expression will be used to represent the fundamental and harmonic currents in the system

$$i(t) = a_0 + \sum_{j=1}^{n} a_j \sin(j\omega t + \phi_j) + v(t)$$

In the equation above, the a_0 is the DC component, a_j, j = 1, 2, 3, … , n represents the amplitude of the jth harmonic at the jth harmonic frequency, ω is the fundamental frequency, ϕ_j is the phase shift of the jth harmonic, and n is the largest harmonic index that is considered. In most general cases there is no DC component, but a_0 was kept in order to account for any sort of DC bias noises. The term $v(t)$ denotes the inherent noise that are embedded in the signal which can be caused by the load or the switching frequency of power electronics devices. In theory, there are an infinite number of harmonic coefficients and harmonic frequencies, but in practice and for the purposes of this paper only a finite set of harmonics are taken into account.

The phase shift of the signal must be taken into account in the estimation. In order to reconstruct an accurate signal estimation with the phase delay, the previous equation will be expanded to the form of:

$$i(t) = a_0 + v(t) + \sum_{j=1}^{n} a_j \cos(\phi_j) \sin(j\omega t) + a_j \sin(\phi_j) \cos(j\omega t)$$

Gradient Algorithm

It is assumed that the phase shift will be constant and independent of time $a_j \cos(\phi_j)$ and $a_j \sin(\phi_j)$ will be constant values. Knowing this, the problem statement will need to be reformatted into a linear representation for parameter estimation [4-6]. This new equation will be

$$i(t) = \varphi^T \theta$$

Where $\varphi \in R^{2n+1}$ and $\theta \in R^{2n+1}$ and are defined as

$$\varphi = [1; \sin(\omega t); \cos(\omega t); \ldots; \sin(n\omega t); \cos(n\omega t)]$$

$$\theta = [a_0; a_1 \cos(\phi_1); a_1 \sin(\phi_1); \ \ldots; a_n \cos(\phi_n); a_n \sin(\phi_n)]$$

Next the vector of unknown parameters is defined by $\hat{\theta}$. The values contained in $\hat{\theta}$ are estimates of θ and still in the form of $a_j \cos(\phi_j)$ and $a_j \sin(\phi_j)$, so both terms must be combined to reconstruct the true values. The process of reconstructing the true values involves the following equations

$$a_j = \sqrt{\theta_{2j}^2 + \theta_{2j+1}^2}$$

$$\phi_j = \arctan(\frac{\theta_{2j}}{\theta_{2j+1}})$$

The reconstructed, or estimated, current signal will be denoted by $\hat{\imath}(t) = \varphi^T \hat{\theta}$. Using the true values and the estimated values of the parameters, the parameter estimate error will be described by $\epsilon = \hat{\theta} - \theta$ [4]. The parameter estimate error can then be used to determine the estimated signal error as follows

$$e(t) = \hat{\imath}(t) - i(t) = \varphi^T(\hat{\theta} - \theta)$$

Now the gradient estimation algorithm will be used to minimize the performing index chosen as e^2. This leads to the gradient updating law of

$$\dot{\hat{\theta}} = -\gamma \frac{\partial e^2}{\partial \hat{\theta}^2} = -2\gamma e \frac{\partial e}{\partial \hat{\theta}} = -\gamma e \varphi$$

In the updating law above the term γ is a constant value larger than 1. For simulation purposes this value was chosen to be 0.01. The gradient algorithm was first tested with the sample signal $s(t) = 2 + 7\sin\left(\omega t + \frac{\pi}{3}\right) + 5\sin\left(2\omega t + \frac{\pi}{4}\right) + 3\sin\left(3\omega t + \frac{\pi}{5}\right) + \ 2\sin(4\omega t + \frac{\pi}{6} + \sin(5\omega t + \frac{\pi}{7})$. The adaptive property of persistent excitation claims that two constant parameters will be guaranteed to converge to their true values for every frequency component in the system [7]. Once the sample signal has been expanded there will be more than enough fundamental frequency components to guarantee that all harmonic amplitudes and phase shifts will converge to their true values. This is shown later in the paper.

After testing the gradient algorithm, some weaknesses were identified. One is the fact that the gradient algorithm will not work without the known fundamental frequency. Generally, the fundamental frequency will be known, but in the case that it isn’t, this poses a large problem for the algorithm. Finally, the dimension of the unknown parameters must be known. This makes it difficult for the user to customize the number of parameters to estimate and the effectiveness of the algorithm.

In order to mitigate the impact of these problems, an alternate method will be introduced to estimate the harmonic parameters. This method will be a robust model based algorithm and it will be able to improve the results of the estimation algorithm without any prior knowledge of the signal or system.

Robust Model Based Learning Algorithm

As mentioned previously the gradient algorithm is not able to estimate the harmonics and reconstruct the signal without some sort of previous system knowledge. By introducing a robust control design, the signal can be reconstructed by the implicit learning of robust control [8-11]. In contrast to any of the gradient based methods, no adaptive law is designed as the robust control asymptotically converges to the unknown values. The robust control design also has an advantage in that it does not require the signal to be periodic. The only requirement for the robust control to be successful is that the derivatives of the function being estimated needs to be bounded.

To design the control needed to estimate the system must first be defined as

$$\dot{x} = -x + \delta_0(t) + u$$

Where x is the state, $\delta_o(t)$ is the unknown time function, and u is the control being designed that will estimate the function $\delta_o(t)$. The magnitude of the time derivatives are as follows

$$|\delta_0(t)| \leq \overline{\delta_0}, \left|\frac{d\delta_0(t)}{dt}\right| \leq \overline{\delta_0}', and \ \left|\frac{d^2\delta_0(t)}{dt^2}\right| \leq \overline{\delta_0}''$$

Next the system is differentiated in order to isolate terms of the control u and the unknown function $\delta_o(t)$. The differentiation leads to

$$\begin{aligned} &\ddot{x} + k_3\dot{x} + k_1x + k_2(\dot{x} + k_3x) \\ &\qquad = \left[\dot{\delta}_0(t) + (k_3 + k_2 - 1)\delta_0(t)\right] + [\dot{u} + (k_3 + k_2 - 1)u + (k_1 + k_2k_3)x \\ &\qquad - (k_3 + k_2 - 1)x] \end{aligned}$$

Where k_1, k_2, and k_3 are the design constants of the controller. Furthermore, the equation can be broken up into a single term, $\eta(t)$, which contains all terms associated with the unknown signal $\delta_0(t)$, and a term v, also known as the control variable, which will be defined as

$$v = \dot{u} + (k_3 + k_2 - 1)u + (k_1 + k_2k_3)x - (k_3 + k_2 - 1)x$$

Choosing a robust controller for v will allow for the controller for u to be solved for through differential equations. A robust controller function can be defined as

$$v(t) = -sign[x]\rho$$

The robust controller functions can be equivocated to

$$\dot{u} + (k_3 + k_2 - 1)u + (k_1 + k_2k_3)x - (k_3 + k_2 - 1)x = -sign[x]\rho$$

The differential equation can now be solved to determine the control u

$$\dot{u} = -sign[x]\rho - (k_3 + k_2 - 1)u - (k_1 + k_2k_3)x + (k_3 + k_2 - 1)x$$

Finally, ρ will be defined as

$$\rho \triangleq \overline{\delta_0}' + |k_3 + k_2 - 1|\overline{\delta_0} + \frac{1}{k_3}\left[\overline{\delta_0}'' + |k_3 + k_2 - 1|\overline{\delta_0}'\right]$$

In analyzing the performance of the learning ability and stability of the robust control a Lyapunov method can be used [8-11]. The Lyapunov function will be

$$V(x,\dot{x},t) = \frac{1}{2}|\dot{x} + k_3 x|^2$$

The above function is positive definite, and it follows that

$$\dot{V} = (\dot{x} + k_3 x)[\eta(t) + v] - k_2|\dot{x} + k_3 x|^2 - k_1 k_3 |x|^2$$

The next step is to integrate both sides which leads to

$$V(t) \leq V(t_0) + \xi(t) - k_2 \int_{t_0}^{t} |\dot{x} + k_3|^2 d\tau - k_1 k_3 \int_{t_0}^{t} |x|^2$$

The function $\xi(t)$ can be defined as follows

$$\xi(t) = \int_{t_0}^{t} (\dot{x} + k_3 x)[\eta(t) + v] d\tau$$

$$= \int_{t_0}^{t} k_3 x[\eta(t) + v] d\tau + \int_{t_0}^{t} \eta(t) dx + \int_{t_0}^{t} v dx$$

$$= \int_{t_0}^{t} x[k_3\eta(t) - \dot{\eta}(t) + k_3 v] d\tau + \eta(t)v(t) - \eta(t_0)v(t_0) - \rho|x(t)| + \rho|x(t_0)|$$

$$\leq \int_{t_0}^{t} \left[\left|k_3\eta(t) - \eta\dot{(t)}\right| - k_3\rho\right]|x| d\tau + \eta(t)x(t) - \eta(t_0)x(t_0) - \rho|x(t)| + \rho|x(t_0)|$$

$$\leq -\eta(t_0)x(t_0) + \rho|x(t_0)|$$

Substituting the expression for $\xi(t)$ into the Lyapunov function, the final expression is

$$V(t) + k_2 \int_{t_0}^{t} |\dot{x} + k_3 x|^2 d\tau + k_1 k_3 \int_{t_0}^{t} |x|^2 d\tau \leq V(t_0) - \eta(t_0)x(t_0) + \rho|x(t_0)|$$

The final expression is bounded by a constant value for all t. It is clear from the full Lyapunov expression that it is bounded by the unknown function and system state at the initial time t_0. Following this equality there are some conclusions that can be drawn. It follows that all $|x|$ and $|\dot{x}|$ are uniformly bounded. In addition to this, it can be concluded from the proposed system and the robust control that both u and $\dot{u}$ are uniformly bounded. Furthermore, if $\ddot{x}$ is uniformly bounded then $\dot{x}$ and x are uniformly continuous. Finally apply the Barbalat lemma to the final Lyapunov expression as $t \to \infty$ the asymptotical convergence of both $\dot{x}$ and x can be confirmed. With all uniformly bounded and continuous variables of the differentiated system using the dynamic robust controller one can observe that it will be globally asymptotically stable.

Following the derivation of the dynamic robust controller, its ability to learn unknown functions has been proven. As $t \to \infty$, both $x, \dot{x} \to 0$ and therefore $u \to \delta_0(t)$. The novelty of the robust controller is that it does not need any sort of prior information about the signal or require the signal to be periodic and can reconstruct the signal with little error.

Simulation Results

Using the normal gradient algorithm, a sample signal was reconstructed. The signal was chosen to be $s(t) = 2 + 7\sin\left(\omega t + \frac{\pi}{3}\right) + 5\sin\left(2\omega t + \frac{\pi}{4}\right) + 3\sin\left(3\omega t + \frac{\pi}{5}\right) + 2\sin(4\omega t + \frac{\pi}{6}) + \sin(5\omega t +$

$\frac{\pi}{7}$) with $\omega = \frac{\pi}{2}$ as mentioned in the previous chapter. So, the gradient algorithm would estimate a DC component and 5 harmonic components.

The normal gradient algorithm will guarantee that the error between the original and estimated signal will be bounded. It is also guaranteed that the parameters will converge to the true values. This is because the signal is of a richness of $2n$+1 degree. This is because each sinusoidal component of the signal contains two frequency components, given by +/-$j\omega, j = 1,2,3, \ldots, n$. For each of the given frequency components it will be possible to estimate 2 of the harmonic components accurately. In this case the error should converge to zero, while the harmonic amplitude and phase shift parameters will converge to their true values. The figures below show the results of the algorithm

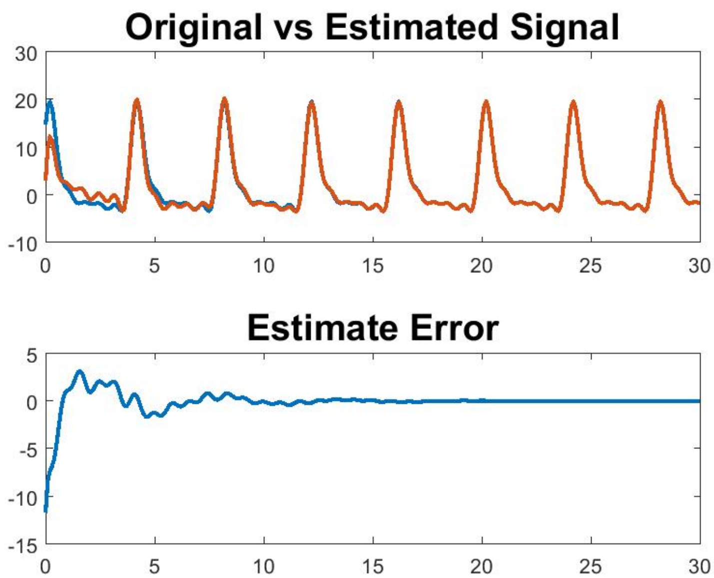

Figure 3: Reconstructed Signal and Estimate Error with Gradient Algorithm

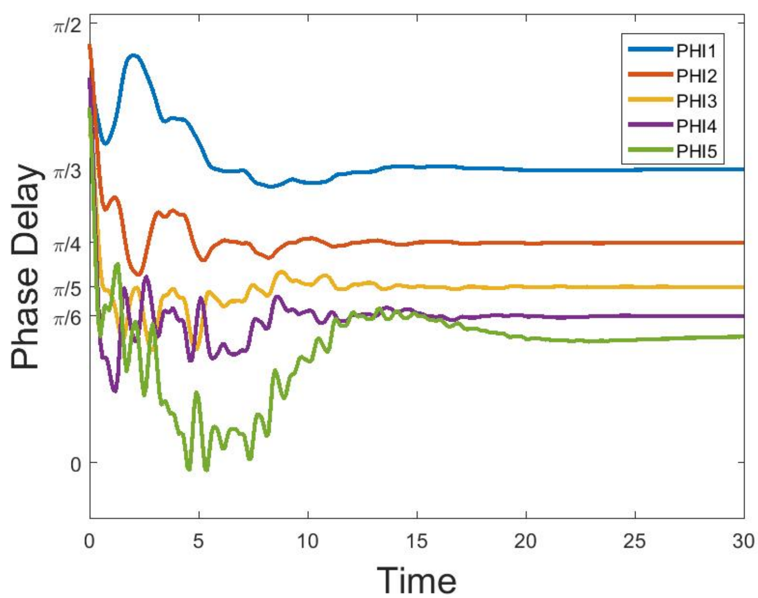

Figure 4: Phase Estimates with Gradient Algorithm

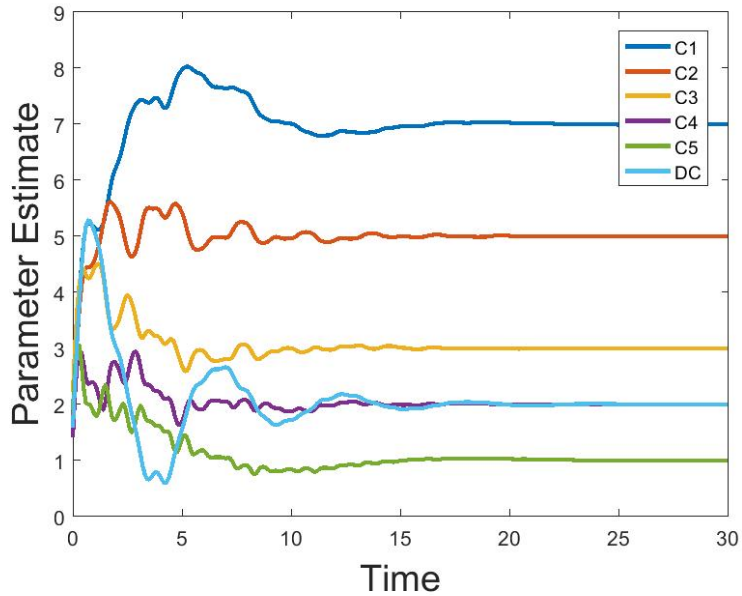

Figure 5: Harmonic Amplitude Estimates with Gradient Algorithm

The initial conditions for the parameter estimates was a vector of random numbers. As can be seen in the figures above the estimated signal is able to catch the original signal within 4 cycles or less than 20 seconds. Also, the DC and harmonic amplitudes converge to 2, 7, 5, 3, 2, and 1 volts as well as the phase shifts converging to the true values.

The final set of results discussed are those of the robust model based leaning control. The main goal of this simulation was to prove that the robust control could learn the signal without any prior data and reconstruct it with extremely little error such that the error signal would practically converge to zero. The sample signal chosen was a simple one with a small fundamental frequency so that the first and second derivatives would be bounded to a smaller value. The signal was also non-periodic to show the robustness of the control. For the first simulation the signal was chosen to be $\delta_0(t) = \sin(\pi * t) + \sin(2\pi * t) + \sin(3\pi * t)$. Therefore, the derivative values would be bounded to 3, 6π, and $6\pi^2$ for the original signal, first derivative, and second derivative respectively. Values for k_i were chosen as 3, .75, and 1.5 as k_1, k_2, and k_3. Below the results for the signal reconstruction and error can be viewed.

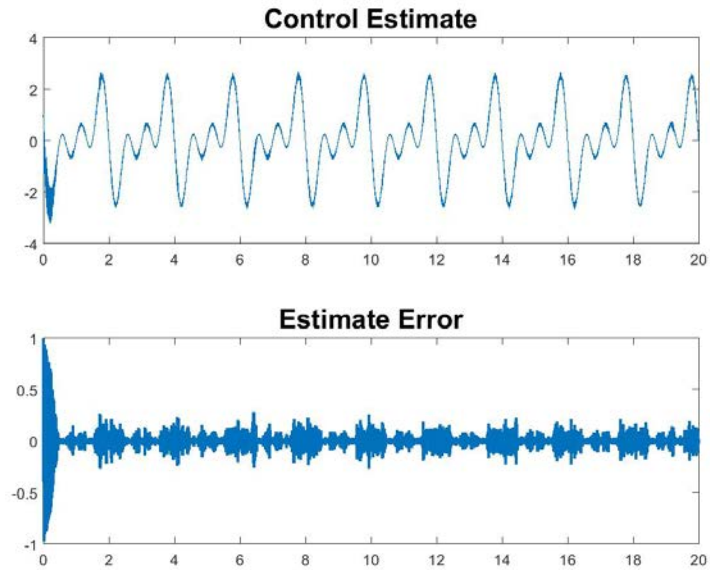

Figure 6: Reconstructed Signal and Estimate Error with Robust Control

It is clear from the above figure that within roughly 1 second, the robust control can learn the function $\delta(t)$ and drive the error to zero. Although this method does not give an extrapolation for each of the harmonic components, it is effective in modeling and reconstructing any type of signal with very little error.

Application

There are many different types of techniques that are used to mitigate harmonics in systems. Though all have proven to be effective, the focus will be on those where the estimation algorithm can be applied and used to either improve or simplify the existing technique.

Passive Mitigation Techniques

Passive techniques are largely used in the industry for harmonic mitigation. They are mainly hardware based and involve installing components that will either impede harmonic current flow or give it an alternate pathway to travel. These include the use of line reactors, multi-pulse converters, and tuned harmonic filters. Techniques such as installing line reactors and multi-pulse converters are simpler to use as they can be installed with little analysis of the system. However, to use tuned harmonic filters there must be some sort of system analysis done, which can be extensive at times [12-14]. To simplify the system analysis, the harmonic estimation algorithms can be applied.

Passive tuned harmonic filters are installed either in parallel or in series and use combinations of inductors, resistances, and capacitors to give harmonics and alternate path and remove them from the system as shown in the figure below.

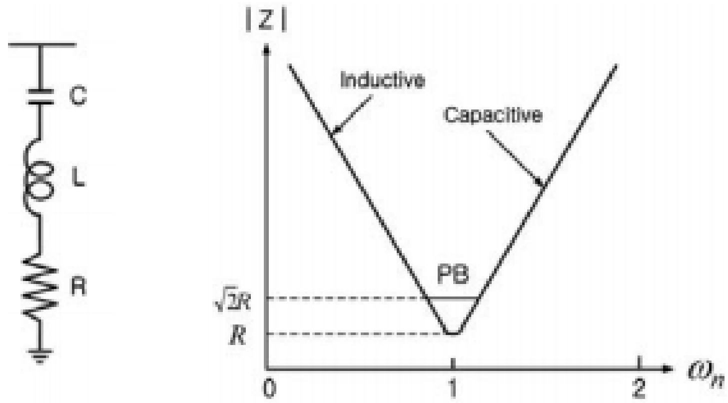

Figure 7: Passive Tuned Harmonic Filter Circuit and Impedance Characteristics

The filter is then tuned to the frequency so that the capacitive and inductive reactances are equal [12]. A passive tuned filter can mitigate the harmonic current of a specific harmonic component. For example, if the user wants to eliminate the 3rd, 5th, and 7th, harmonic values then a separate filter will be designed for each one. Since the reactance of the capacitor and inductor are the same, the tuning frequency of the harmonic is chosen using the following equation.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where f_r is the tuning frequency of the filter, L is the inductance of the filter, and C is the capacitance of the filter. It is important to note that the sizing of the filter components must be taken into consideration depending on the amplitude of the harmonic current.

As mentioned before the process to design a passive filter requires extensive calculation and computer modeling of the system to identify the harmonics in the system and the level of intensity they are at. This is where the harmonic estimation algorithm can be applied. With the algorithm the user can successfully identify what kind of harmonics are in the system and how large they are. After estimating the harmonic components, the user can then determine which ones are to be mitigated and the filters can be designed for each one.

Active Mitigation Techniques

Another method of harmonic mitigation is known as active harmonic filtering. This method works by injecting an equal but opposite current or voltage into the system at the point of common coupling and this effectively cancels out the existing harmonic content. Active harmonic mitigation uses either voltage source inverters or current source inverters which use fast switching insulated gate bipolar transistors (IGBTs) to produce the current needed to cancel the harmonic components of the system [13-14]. Active harmonic filters are generally used on location for loads that are nonlinear. Companies like Schneider Electric and ABB have a variety of active harmonic filter models available. Active filters are usually installed in parallel or in series, although the parallel set up is the most common as depicted in the following figure.

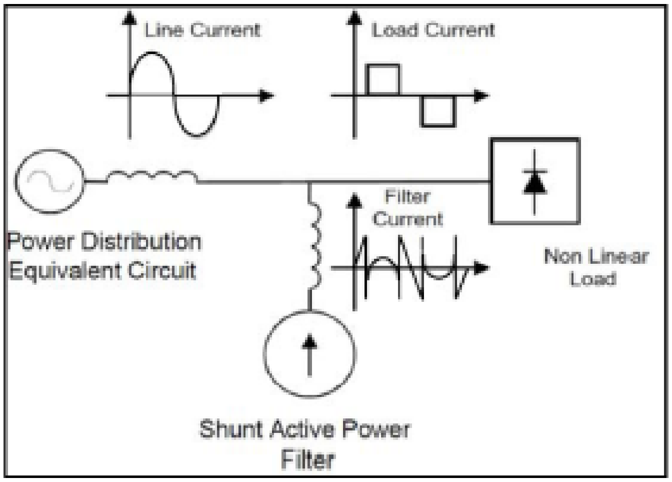

Figure 8: Active Harmonic Filter and Waveform

Active harmonic filtering provides another opportunity to apply the harmonic estimation algorithm. The algorithms, specially the robust control algorithm, can be used in the control section of the active filter to quickly and accurately estimate the complete signal. The fundamental portion of the signal can then be extracted, which will leave only the harmonic components. The remaining harmonic components can then be changed to an opposite polarity and injected into the system to negate all harmonics.

Conclusion

As technology becomes more advanced and more nonlinear devices are being used on power systems there is an increase on harmonics presented on power systems. In order to maintain power quality and high power factor, harmonic mitigation methods must be developed and implemented. There are many different types of harmonic mitigation techniques that are being used today. The most common of these techniques are passive and active filtering. Passive filtering involves extensive system analysis to design a bandpass filter that will remove harmonics that the filter was specifically designed for. Active filtering extracts the harmonic components from the system current and uses IGBTs to generate an equal and opposite harmonic current to cancel out the existing harmonics on the system. For both techniques to be used the harmonics in the system must be identified and this can be a difficult task due to system variations such as changing load and varying power electronic devices. In this paper, two methods for harmonic estimation were derived and discussed. Simulations were run with sample signals to analyze the performance of the algorithms and possible applications were determined for each algorithm based on harmonic mitigation methods that were previously discussed.

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